

Contents 1968

1968-1

Notes on representations of semi-simple Lie groups.

1968-2

January 3: On G -manifolds and G -bundles. Candidates for equivariant bordism theory.

January 4: Equivariant cobordism theory.

January 10: Notes on Chern classes. Cohomology of a blow-up. Proving $K_G(\mathbb{P}V) \xrightarrow{\sim} K_G(X)/(\lambda_{-T}(V))$.

1968-3

January 22: Reorganization of results on representations of semi-simple Lie groups. Attempts at Nakayama's lemma. Iwasawa formulas. Principal series. Borel-Weil problem.

1968-4

Late January: Concerning Nakayama's lemma. Mackey's double coset formula.

1968-5

February 1: Summary,

February 5: Summary of Bruhat 2. Intertwining morphisms. Definition of principal series. Maximal ideals in $U(\mathfrak{g})$.

1968-6

February 8-12: Harish-Chandra's result in irreducibility of \mathfrak{g} , \mathfrak{k} modules and related work.

1968-7

Continuing working on the conjecture that $U(\mathfrak{g}) \otimes_b \lambda$ is irreducible \Leftrightarrow for no $\alpha \in \Sigma$ is $\frac{2\lambda(H_\alpha)}{\alpha(H_\alpha)}$ an integer ≥ 0 .

February 13: Suppose that $\lambda(H_\alpha)$ is an integer ≥ 0 whenever α is a simple root, then $U(\mathfrak{g}) \otimes_b \lambda$ is reducible. Canonical isomorphism $\gamma : U(\mathfrak{g})^\mathfrak{g} \rightarrow U(\mathfrak{h})^W$.

February 14: Talk on irreducible modules over enveloping algebras.

February 15: On irreducible weight representations. Calculations for $sl(3)$.

February 19,20: Summary of preceding 2 months work on Lie algebras.

February 21: Harish-Chandra theorem.

February 22: Some conjectures.

1968-8

A summary of Rallis's thesis.

Notes on a talk by Steinberg: Galois cohomology of algebraic linear groups. J.Wolf colloquium on complex flag manifolds.

February 23: Notes from Zelobenko, Dokl. 7, 1403 and calculations for $sl(2, \mathbb{R})$. Class 1 representations.

February 24: On two operations

1) if ϕ is a function on G such that $\phi(kan) = \nu(a)$, $\hat{\phi}(g) = \int \phi(gk)dk$;

2) if $f(kgk^{-1}) = f(g)$, $F_f(a) = e^{\rho \log(a)} \int_N f(an)dn$

and spherical functions.

February 25: Ideas for further work.

1986-9

MIT Topology seminar talk by Ralf Reid: Isotopy and embedding of polyhedra in euclidean spaces, the deleted join and π -manifolds as criteria.

Notes on material of Kamber-Tondeur. Wall's formulation of Swan's theory.

April 6,7: Cobordism classification. Typed notes on the representability of various functors. Classification of $\mathbb{C}P^{n-1}$ bundles over a simply connected space. Milnor's universal bundles.

April 8: Relationship between Milnor's G -construction and Alexander-Spanier cochains.

April 9: Summary of work since April 1.

April 15: To prove a Verdier duality theorem for simplicial sets.

April 16, 20: Amitsur complex as a cobar construction. Fredholm theory review. Fourier transforms on manifolds.

April 22: Attempt at a parametrix proof of the Atiyah-Singer theorem.

April 26: Summary of problems worked on since April 1. Conjectures.

1968-10

May '68: Summary of work on the Atiyah-Singer Index theorem and various short notes including some on a Chern class for a Weyl algebra, Kolmogorov-Arnold stability theorem, algebraic models for manifold theory, algebraic cobordism theory.

1968-11

Frank Adams 1,2,3,4,5,6. Summary of results of Adams and others working on homology and cohomology with a view to generalization.

Bordism theory: a preliminary version of an article. Summary of work on cobordism theory done in July 1968 at Battelle.

July 25: Conversation with Browder.

1968-12

November 8,10,18: On simple characteristics. Statistical mechanics after Mackey. The theory of Freyd.

1968-13

Fall '68. §1 Fundamental groupoids, singular complexes, local coefficient systems. §2 $D_{lc}(\mathcal{C})$ and Whitehead theorem. §3 Review of work of Andre. §4 Every simplicial set is homotopy equivalent to the singular complex of a category.

December 10: Construction of the category of motives over a point.

December 11: Thom transversality theory. Characteristic classes and motives. Ideas on motives.

December 16: Sullivan's bordism theories using varieties with prescribed singularities. Determination of motive category.

December 18: Problem: To what extent is a homology theory with products on a category of smooth manifolds related to a multiplicative cohomology theory á la Atiyah-Hirzebruch on the category of finite simplicial complexes.

1968-14

December 24: More motives. Motives over a base manifold.

December 25: Local cohomology in cobordism theory.

December 26: Motives over a base manifold.

December 28: More motives.

December 31: More motives. Motives over a base manifold.

1968-15

Mixed notes and a preliminary version of a paper on higher algebraic K -theory, including the definitions $K_i(A) = \pi_i(BGL(A)^+)$, $i \geq 1$ and $\tilde{K}(X, A) = [X, BGL(A)^+]$.

1968-16

Preliminary version of a paper on algebraic K -theory.

Contents 1969

1969-1

January 12: Note on $Q_n = \mathbb{P}(1 + \mathcal{O}(1)/\mathbb{P}^n)$. Calculation of Chern numbers for Q_n . Description of where an old calculation went wrong.

January 13: Riemann-Roch formula.

January 18-20: On G -manifolds where G is a compact Lie group. Steenrod operations in cobordism. Character as a natural transformation on cohomology theories. Steenrod operations on complex projective spaces.

January 25: Witt ring reviewed.

January 27-28: On $\Omega(\mathbb{P}E)$. Formulas for $K_G(\mathbb{P}V)$. Complex cobordism ring.

January 29: On $\mathcal{F}(A) = \{F \in A[[X, Y]] : F(X, 0) = F(0, X) = X, F(X, F(Y, Z)) = F(F(X, Y), Z)\}$. Newton's formulas. Determination of $\Omega_{\mathbb{Q}}(\text{pt})$.

January 30: On a ring V with a formal group law and $V(X)$, a cohomology theory on the category of manifolds with values in V -algebras and endowed with a Gysin homomorphism.

1969-2

February 1: Chern numbers map $\Omega(\text{pt}) \rightarrow H_*(BU, \mathbb{Z})$. On the theorems of Cartier on curves in formal groups.

February 2: On equivariant bordism theory.

February 3: Operations in cobordism theory (after Novikov and Adams). Thom isomorphism and Novikov operations. How to invert a power series.

February 5: Characteristic numbers (revisited). Stong-Hattori theorem.

February 6: Stong-Hattori theorem. Group law over $\Omega(\text{pt})$ determined by $c_1(L_1 \otimes L_2) = F(c_1(L_1), c_1(L_2))$. Formulas. Conner-Floyd version of Stong's calculation.

February 9: Cartier's theory of curves in formal groups.

February 10: Applications of typical formal group laws on cobordism theories.

February 12: Formal group law calculations. Effect of homotheties on cobordism. Outline of paper on cobordism.

1969-3

Material for a paper called *Cocordism and Formal Groups* and a draft of a letter to Cartier.

February 19: Equivariant cobordism theory Ω_G .

February 22: Remarks toward a cobordism theory in algebraic geometry.

February 25: Equivariant cobordism and formal group laws.

1969-4

March 2-5: On cohomology operations. Determination of stable homotopy over \mathbb{F}_p .

Lemma: $\text{res} \begin{bmatrix} fw \\ fg \end{bmatrix} = \text{res} \begin{bmatrix} w \\ g \end{bmatrix}$.

March 7-9: Review of formulas for $\Omega(\mathbb{P}E^\vee)$. Residue in dimension 1.

March 8: Residues in dimension 1 again. The push forward map F_* for projective bundles.

March 10: Stong-Hattori from rigidity of type 1 laws. A theorem of Adams. Conner -Floyd theorem. A note clarifying an apparent paradox.

March 10: Characterizing cohomology theories by their formal group laws. On $F(X, Y) = X + Y + XY$. Brown-Pedersen theory. Riemann-Roch lemma.

March 10: On $F(X, Y) = X + Y + XY$.

1969-5

March 12: Decomposition of a Lazard scheme over \mathbb{F}_p . Problem sheet. Spectral sequences result.

March 17: Review of local cohomology theory. Proof of Thom isomorphism.

1969-6

Rough notes exploring cobordism with supports.

1969-7

March 17: Cobordism with supports.

March 19: Typical group laws.

March 19: Some calculations of typical coordinate changes.

March 23: Relations between Chern classes in cobordism theory and geometric Chern classes.

March 27: Operations in generalized cohomology theories. Notes on Andre Weil's talk in Bourbaki 312, June '66. Program for equivariant cobordism theory. Some rough notes.

1969-8

April 14,15: Operations in ΩT .

April 17: Notes on category schemes from Fall '68. Unsolved problems.

1969-9

Rough notes on Witt rings, Bochestein and Steenrod operations.

1969-10

April 22: On $K_2(F)$ where F is a field. Formal category schemes.

April 27: On category schemes, §1 A, A algebras and bialgebras.

1969-11

May 5: A chapter §2 Formal categories over a field. Formal category schemes.

May 6: A letter to Serre.

May 8: Generalities on category objects.

May 9: Pseudogroups versus formal groupoids. Remarks on DR Chern classes. Lecture plan. Affine categories and operations in generalized cohomology theories.

May 21: Affine categories and operations in generalized cohomology theories.

May 14: Given a category, find an appropriate category of modules. Program of research on cobordism theory.

1969-12

May 16: Cobordism modulo torsion in algebraic geometry. Formal group law. Residue formula. Excision in $\Omega_{\mathbb{Q}}$.

May 21: Affine categories and operations in generalized cohomology theories.

May 22: Operations in generalized cohomology theories: §1 Dicogebbras, §2 Dibigebbras, §3 Operations, §4 Multiplicative operations.

May 25 and June 11: Unoriented cobordism and formal laws: §1 Formal group laws of height ∞ in characteristic p , §2 Unoriented cobordism theory.

1969-13

June 3: New proof of a composition theorem.

June 5-7: Characteristic numbers. K^* -theory operations. Stong-Hattori theorem as formulated by Adams. The general picture about characteristic numbers and Wu relations.

June 11: The cobordism class of a blow-up.

1969-14

Obselete June 11: Decomposition theorem $\Omega = L \otimes_{LT} BP$ where $BP = LT \otimes_L \Omega_*$.

June 12: Operations in cohomology theory and operations in ΩT , §1 Calculation of Aut^\otimes for Ω and ΩT .

June 12: Some basic geometry. Conversations with Wall. Tate lecture.

June 13: Arbeitstagung: Kuiper's talk on Kirby's solution of the annulus conjecture; review of Griffith's lecture on algebraic cycles; comments on Adam's lecture on Quillen's work.

June 19: Notes on Wall's lecture on Kirby-Siebenmann.

1969-15

July 1,14: The Connor-Floyd theorem in equivariant bordism.

July 4: Adams operations in Ω .

July 5: On K -theory characteristic numbers, §1 Formal groups of height 1.

July 6-14: Real K -theory and cobordism theory. Spectral representation of ΩR .

July 21: More on an action of a compact group G on a manifold.

July 24: On the universal nature of cobordism theories.

1969-16

August 7: Concerning group G having a representation V such that SV is G -free.

August 12: General facts about G -manifolds, G finite, odd order.

August 19: Equivariant cobordism revisited.

August 25: Analysis of a Gysin sequence.

August 27: Multiplicative operations $\psi^k : U^{\text{ev}} \rightarrow U^{\text{ev}}(X)[\frac{1}{k}]$. An error recognised.

August 30: Impossibility of defining an integral ψ^2 on U^{ev} by Steenrod's methods.

1969-17

Typed paper: Voisinages tubulaires dans a topos localement annelés.

Notes on a talk by May.

1969-18

September 1: Clean intersections.

September 5: Direct construction of a map $U_G(X) \xrightarrow{\phi} RG \otimes (\mathbb{Z} \otimes_L U(X^G))$. Representations of the symmetric group.

September 8: Formula for $Q : U^{\text{ev}}(B\mathbb{Z}_2 \times X)$.

September 10: Induction and restriction formulas for equivariant cobordism.

September 12: Total Steenrod operations.

September 16: Roots of orientations.

September 17: On $SO^*(X)_{[\frac{1}{2}]}$.

September 18: Symplectic theory.

September 21: Symplectic and oriented cobordism off 2.

September 22: SU bordism off 2.

September 24: Isomorphisms between multiplicative characteristic classes for SU bundles and cocycles.

September 25: On the localization theorem of Tom Dieck.

September 26: Oriented cobordism.

1969-19

October 6: More on norms and traces.

October 7: Notes on orientations.

October 8: Norm of a double covering.

October 9: On signs.

October 13: How to modify U_G to get Chern theory.

October 14: Some examples.

October 15: Dimension (in the sense of Atiyah) of a compact Lie group. Remarks of Atiyah on the Completion theorem.

October 17: Complex orientation of a map.

October 18: On H_G^* .

October 20: Spectrum of $H_G^{\text{ev}}(\text{pt})$, equivariant cohomology with coefficients in \mathbb{Z}_2 .

October 24: Invariant prime ideals, $I_G(X)$, in $H_G^*(X)$.

October 25: On proving $J_G(X) \xrightarrow{\sim} I_G(X)$.

October 27: Proof that $J_G(X) \xrightarrow{\sim} I_G(X)$ when G is a torus. Localization theorem.

October 28: Fixed submanifold of a manifold with a \mathbb{Z}_p action.

1969-20

November 5,7: On the ring homomorphism $H_G(X) \rightarrow \varprojlim_{(A,\lambda)} H_G(\text{pt})$.

November 29: On the symmetric group $\sum(n)$ and Dickson's theorem.

Various notes on H_G .

1969-21

December 9: On PL-cobordism plus various rough notes.

December 15: Localization at fixed points and power operations in K -theory.

December 15: On symplectic cobordism. On Hirzebruch's proof that $\mathbb{H}P^1$ is not weakly complex. Future projects.

Problems to give students. On higher algebraic K -theory.

December 23: On the symmetric group.

December 26: Power operation $P : H^q(X) \rightarrow \prod'_{k \geq 1} H^{kq}(B\Sigma_k \times X)$.

December 29: On $F(V) = \prod'_{k \geq 1} H^*(B\Sigma_k, V^{\otimes k})$, coefficients in \mathbb{Z}_2 .

December 27: On the Thom homomorphism and unoriented symplectic cobordism.

December 31: Nakaoka's calculation of $R = \mathbb{Z}_2 + \oplus_{k \geq 1} H_*(B\Sigma_k)$, coefficients mod 2.

1969-22

Notes on induced representations as given by Bruhat.

1969-23

Notes on Mumford's Bowdoin lectures on abelian varieties.

1969-24

More notes on Mumford's lectures. Notes on Tate's lectures. Notes on S -theory for groups. Kleinman on Pic.

1969-25

Notes on work of Lambek.

1969-26

Rough notes on ergodic theory and results of Shub and Anasov. Notes on Atiyah and Bott's elliptic complexes.

1969-27

List of important papers on K -theory. Further rough notes on various topics.

1969-28

Further rough notes including some on expanding maps, entropy, Moser's proof of structural stability and Stable manifold theorem

1969-29

Notes on work of Smale. Copy of a paper by Epstein and Shub. Notes on entropy. Mackey's presentation of statistical mechanics.

Contents 1970

1970-1

Jan 5,8: Dyer Lashof operations.

1970-2

Starts with Miscellaneous notes: outline of paper on cobordism theory, equivariant K-theory, power operations, ζ function of a scheme of finite type over \mathbb{F}_q .

Jan 9: Symplectic cobordism.

Jan 11: Generalized cohomology of real projective space. L-Novikov operations.

Jan 14: Symplectic cobordism. Atiyah's lectures on cohomology operations.

Jan 16: Symplectic cobordism. Notes on KO.

1970-3

$\oplus_{n \geq 0} H_*(BU(n))$.

Jan 17, 18, 20: $H^*(GL_n(\mathbb{F}_q), \mathbb{Z}_l)$, $(l, q) = 1$.

Jan 21: Invariants under the symmetric group of $\Lambda[x_1, x_2, \dots, x_n] \otimes S[y_1, y_2, \dots, y_n]$.

Jan 24: Remarks on $H^*(BGL(\mathbb{F}_q), \mathbb{Z}_l)$. Dyer Lashof operations on $H^*(X)$ where X is an infinite loop space.

Summary of open problems related to work on the Adams's Conjecture.

1970-4

Jan 25: $H^*(B\Sigma_n, \mathbb{Z}_p)$, p odd.

Jan 26: Formula for $w_t(f_*E)$ where f is a double covering. Invariants in $\Lambda V^* \otimes S^*$.

Jan 27: $H^*(B\Sigma_n, \mathbb{Z}_p)$, p odd. Review of Milnor's description of the dual of the Steenrod algebra

Jan 29, 30, 31, Feb 2: Steenrod operations for p odd. Multiplicative property of norms.

Feb 3: Related conjectures.

1970-5

Feb 4: Test conjecture regarding the multiplicative property of a universal natural transformation.

Feb 6: Chow ring of G/B where G is a connected, reductive algebraic group over \mathbb{F}_q .

Feb 8: Spectral sequence for (G, EG, BG) .

Feb 9: Is $H^*(BG(k)) \rightarrow H^*(BT(k))$ injective?

Feb 10: Proposal for a definition of algebraic K -theory.

Feb 12: Conjecture about when $GL_n(R) \rightarrow GL_n(k)$ induces an isomorphism modulo ℓ .

Feb 16, 17: $\coprod_{n > 0} \text{Spec } H^*(B\Sigma_n, \mathbb{Z}_p)$ and above conjecture.

Feb 18: Boardman's Theorem on the construction of connective generalised cohomology theories for symmetric H -spaces. Sullivan's claim that BU^\otimes is not isomorphic to ΩX where X is a space with finite k invariants.

Feb 19: To understand Brauer theory from the point of view of representation rings.

Feb 19: Let G be a finite group and k a field, $\text{char } k \neq 2$ such that all $k[G]$ -simple modules are absolutely reducible. Then

$$R_k(G) \xrightarrow{\psi^{-1}-1} R_k(G) \rightarrow RO_k(g) \oplus RSp_k(G) \rightarrow R_k(G) \xrightarrow{\psi^{-1}-1} R_k(G)$$

is exact.

Sullivan's suggestion to construct basic virtual representation of $GL_n(\mathbb{F}_q)$.

Various details related to the Adam's conjecture in the real case.

Feb 23, 24: To check that $RO_k(G) \oplus RSp_k(G)$ is a λ -ring in such a way that the forgetful functor to $R_k(G)$ is a λ -ring homomorphism.

The Boardman-Vogt Theorem. Mumford's conjecture.

Feb 24: Projects (state of):

- (a) Spectrum of an equivariant cohomology ring.
- (b) Adam's conjecture.
- (c) Cohomology of $BG_k(\mathbb{F}_q)$.
- (d) Symmetric groups.
- (e) Boardman-Vogt theorem and KADL theory.
- (f) Higher K -theory.

1970-6

Homotopy Axiom. Symplectic bordism

March 1: Let $U_n \rightarrow P \rightarrow X$ be a principal bundle. Then $h(X) \rightarrow h(P \times_{U_n} U_n/N_n)$ is injective onto a direct summand for any generalised cohomology theory with finite coefficients.

March 2: Segal's localization for non-simply connected spaces.

March 5: Localization and higher algebraic K -theory (ctd).

March 7: Cohomology mod ℓ of the group of rational points of a non-split torus.

March 8: Geometric Frobenius. Detecting $H^*(BG(\mathbb{F}_q))$.

March 9: $H^*(BG, \mathbb{Z}_\ell) \xrightarrow{\sim} \lim_{\rightarrow n} H^*(BG(\mathbb{F}_n), \mathbb{Z}_\ell)$.

March 11,12: Higher order K -theory.

March 13: If ℓ is prime and G an ℓ -local group then $H^Q(BG, \mathbb{Z}_\ell) = 0$.

March 14: The universal map $k(X, R) \xrightarrow{\gamma} K(XR)$ is representable.

March 15: $K_*(\mathbb{Z})$.

March 18: Algebraic K -theory.

1970-7 Brauer lifting of standard representations of $GL(\mathbb{F}_q)$.

March 23: Stable homotopy of symmetric groups. Comology of finite groups of rational points.

March 26: Representability of algebraic K -theory.

March 28: Derivation of spectral sequence

$$E_2^{pq} = H^p(BG, H^q(G/H)) \Rightarrow H^{p+q}(H)$$

.

Equivariant comology and etale cohomology. Outline of paper. Cohomology rings of groups.

1970-8

April 2; Exterior generators of $H^*(BG_\ell)$. Determination of the mod ℓ cohomology of the general linear and symmetric groups with entries in a function field of characteristic $p \neq \ell$. Localization theory. The operator ϕ . Draft for paper on the cohomology of the finite groups of rational points.

April 4: Eilenberg-Moore approach to the cohomology of finite groups of rational points.

1970-9

The map $f : EG \times_G (BG)^c \rightarrow BG$.

April 11: More K -theory, arithmetic Chern classes.

April 12: Arithmetic Chern classes and algebraic K -theory.

April 14: Computation of ϕ for a torus.

April 16: Formulation of the derivation property of ϕ .

April 18: Computation of ϕ .

April 19: Torsors, extensions and cohomology.

April 20: More on ϕ .

April 25: Parameter topos scheme for K_* .

1970-10 Rough notes and outline of a book on cobordism including material on transversality and Whitney embedding theorem.

1970-11

More rough notes on cobordism

May 5,7: Notes on computation problems in the cohomology of groups.

May 11: Heuristic calculation of $K_i(\mathbb{F}_q)$.

March 14: Formulation of representing a fibre-bundle theory.

March 15: Computation of some K_1 's.

May 28: On algebraic K -theory.

1970-12

Various rough notes.

June 16: Exact sequences in K -theory.

June 18: How much of a cohomology theory can be recovered from knowing it on the spaces BG .

June 24: A 2-category approach to homotopy theory.

June 26: Grand scheme for algebraic K -theory. Weil-Tate conjectures.

June 28: More evidence that $H_*(GL(\mathbb{F}_q), \mathbb{Z}_p) = 0$.

1970-12.5 Paper: K -theory associated to a finite field, I

1970-13

July 5: Cohomology of Sylow 2-subgroups of $GL_q(\mathbb{F}_{2^d})$.

July 10: Cohomology of $N(\mathbb{F}_{2^d})$.

July 19: Sullivan's Theorem.

July 28: λ -ring structure on $K^0(X, A)$.

1970-14

August 11: The immeuble for Σ_k .

August 22: The Tits complex of $GL_n(k)$.

1970-15

September 20: Let G be a group in a topos \mathcal{T} . When is the cohomology of the discrete group $\Gamma(G)$ the same as the cohomology of \mathcal{T}_G ? Equivariant cohomology.

September 24: F -isomorphism theorem for compact G -spaces and application to a theorem of Serre.

September 28: Euler characteristics of G -spaces.

1970-16

October 1: $f_! : K_G(X) \rightarrow K_G(\text{pt})$ where G is a finite group and X a compact analytic manifold.

October 3: Sullivan's Stiefel-Whitney classes.

October 4: Inverse images of harmonic functions.

October 9: Localization at a maximal elementary abelian subgroup.

October 13: On Thom's theorem realizing rational classes, exponential map for $GL_n(\mathbb{C})$, Bott cocycles.

October 14: Funny unitary group. To understand $[X, S^3] \rightarrow H_{\text{DR}}^3(X)$.

October 16: Bott's formula for Chern classes.

October 24: On symmetric products.

1970-17

Nov 4: Attempt to see which characteristic classes of $GL_\infty(G)$ are detected by finite abelian ℓ -groups.

Nov 5: $R_\Lambda(G)$.

Nov 13-14: Towards an understanding of Thompson's Theorem. Normal p -complements.

Nov 16: Cohomology theories of symmetric product types.

Nov 26: K of a local field, continued.

Nov 27: Characteristic classes of finite groups.

Nov 28: K of a local field.

Nov 29: If A is a strictly local ring then $H^*(GL(A), \mathbb{Z}/\ell\mathbb{Z}) = (\mathbb{Z}/\ell\mathbb{Z})[c_1, c_2, \dots]$.

Nov 30: Theorems on group theory.

1970-18

Dec 4: Material on Tate conjecture and normal p -complements.

Dec 5: Haefliger's classifying space.

Dec 11: On Haefliger's structures.

Dec 15: Computation of $\max [\ell]$ -subgroups in $GL_n(\Lambda)$ where $\mu_\ell \in \Lambda$, $\ell^{-1} \in \Lambda$ and Λ is a Dedekind domain.

Dec 17: To compute mod ℓ cohomology of $SL_2(K)$ using Serre's trees.

Dec 20: On p -group with $\Omega_1 \subset Z(P)$.

Dec 25: Localization theorems.

Dec 27: $H_*(GL(\Lambda))$.

Dec 28, 30: Central $[p]$ -groups.

1970-19

Miscellaneous notes relating to 1970 material

Contents 1971

1971-1

January 1, 2, 3: More on p -groups.

January 12: K -groups for a curve over a finite field.

January 15: Cohomology of $GL_n(\Lambda)$ where Λ is a discrete valuation ring with $[\Lambda : \mathbb{Z}_p] < \infty$.

January 17: Spectrum of $SL_2(F)$, $[F : \mathbb{Q}_p] < \infty$.

January 24: Notes on a trip to Atlantic City and Institute. Theorem of Borel.

1971-2

February 3: Deformation of Torsors paper.

February 3,4: Status of stability theorem. Theorem : Let Λ be a perfect ring of integers. Then $K_i(\Lambda)$ is uniquely p divisible for $i > 0$.

February 5: Computation of the differential d . On the stability range. Monoidal categories. Seattle talk: Finite generation of K_i for rings of integers.

February 17: Calculation of $R_G(X)$ where X is a curve over a finite field k and G has finite order prime to $p = \text{char}(k)$.

February 20, Projects. Conjectures about $K_a(X)$, X a curve over \mathbb{F}_q .

February 21: Conjectures about $K_a(X)$, X a curve over \mathbb{F}_q . Projects.

February 22: Review of Mather's theorem. Problem on elements of $[BG, BGL(A)^+]$ where G is a simplicial group.

February 26 K - groups globally.

February 28: Conjecture on Real K -theory.

February 27: Stability Theorem. Notes: category of stable trivializations of (P, Q) or stable isomorphisms of P and Q ; Lichtenbaum letter; acyclicity of the map $f : \underline{J} \rightarrow \Gamma$; outline of proof of cohomological Mather theorem.

1971-3

March 1: Check of conjecture against Tate's result for K_2 . Conjecture: If C is a Dedekind subring of a number field F then $K_{\text{odd}}(C) \rightarrow K_{\text{odd}}(F)$ in $\dim > 1$.

March 6, 8, 14: How Mather deloops BG when G consists of orientation preserving diffeomorphisms of \mathbb{R}^k .

March 10: Stasheff classification theorem.

March 12: Principal bundles for a topological groupoid. On Haefliger structures,

March 13: Review of proof of classification theorem for principal G - manifolds.

March 14: Mather's theorem on diffeomorphisms with compact support of \mathbb{R}^n .

March 15, 18: More Mather.

1971-4

March 20, 22: For a small category \mathcal{C} , to define \mathcal{C} -bundles over a topos and prove a classifying theorem.

March 26: Summary of problems and progress.

March 30,31: \mathcal{C} -torsors and initial understanding of Mather's theorem.

1971-5

April 1: Pull back of torsors in Topos \mathcal{T} . Mather's theorem for general stacks.

April 2: Categorical aspects of above.

April 3: Classifying torsors.

April 4: Lubkin coverings and \mathcal{C} -torsors when \mathcal{C} is the category associated to a partially ordered set.

April 5 : When are the representable \mathcal{C} -torsors dense homotopically in \mathcal{C} -torsors?

April 6: On the category Δ of ordered sets $[n] = \{0, 1, \dots, n\}$ and weakly increasing maps.

April 7: Colombia talk on the K -theory of rings.

April 11 False hope and counter example.

April 12: Exact sequences in K -theory problem,

April 14 Fibred topos.

April 24: K -theory.

April 26 When is BG the 'homotopy-theoretic' limit:

$$\lim_{\substack{\longrightarrow \\ P \in \mathcal{P}}} P/G = BG,$$

where \mathcal{P} is the category of G -spaces.

April 28: Eilenberg-Zilber theorem.

1971-6

Conjectures on algebraic cycles. On the derivation of etale cohomology from K -theory. Generalized Jacobians.

1971-7

May 1: K -theory. The category \mathcal{E} of finitely generated projective R -modules.

May 3: Category of torsors of \mathcal{C} .

May 5: Proof that \mathcal{E} has the homotopy type of $B\Sigma_\infty^+$.

March 7: Model for $BGL(R)^+$.

May 13,14: K -theory. New way of defining K_0 for small abelian categories.

May 16: Attempt at exact sequence for K -theory.

May 17: Mather's theorem - sheaf theoretic point of view.

May 21,23: More on Mather's theorem.

1971-8

June 2: On group completion.

June 4: Galois cohomology. Diversion on localization.

July 8: Quasi-fibration. Summary of Friedlander's thesis.

1971-9

Notes for a paper on K -theory

1971-10

June 6: Show that the commutator subgroup (G, G) is perfect when G is the group of diffeomorphisms of compact support on \mathbb{R} .

June 12: Kummer theory.

June 14: Summary of work on K -theory. Idea about J -homomorphism.

June 24: On category finitely generated projective projective R -modules.

June 27,28: The Artin-Mazur simplicial set. Let \mathcal{B} be the bicategory and X the associated bisimplicial set, then $AM(X)$ has the homotopy type of $BGL(R)^+$. Acyclicity of the map $f : \underline{J} \rightarrow \Gamma$. Haefliger structures. Philips-Gromov theorem. The Lie algebra of vector fields with compact support on \mathbb{R} . Mather theorem. Theorem of Milnor.

July 9: Friedlander's thesis. Stability. $BGL(A)^+$ as an infinite loop space.

1971-11

August 4: Relation between C^\wedge and $C^{o\wedge}$

August 5: Sheaves associated to a simplicial complex. Grothendieck's filtration.

August 6: Simplicial sheaves over simplicial sets.

August 7: Homotopy $D_{\ell c}(X)$ (complexes whose homology sheaves $\mathcal{H}^\ell(K)$ are locally constant). Acyclic maps in homotopy theory. Classification of acyclic maps. Cohomology of loop spaces.

August 8: The stability problem.

August 9: Stability. Existence of a stable range for symmetric groups.

August 18: On $H^*(GL_3(\mathbb{F}_2), \mathbb{Z}/2)$.

August 24: Review of Thompson's theorems.

1971-12

September 3: Cancellation.

September 4: Serre's stability theorem.

September 5: Acyclic maps. Killing perfect subgroup of fundamental group.

September 14: Stability and Reid's proposition.

September 16: Serre's theorem.

September 20: $VF = FV = p^d$,

September 25: Serre's theorem. Burnside ring.

September 29: Theorem of Kaloujnine. Tate's theorem

October 3: Cohomology theories and Σ_n .

October 18: More on Lang's theorem.

October 21: Summary of problems: group completion theorem; good point of view; stable splitting of exact sequences; configurations and loop spaces. Cornell talk: 'K-theory of discrete rings'.

October 27: Understanding buildings. Conjecture.

1971-13

November 1: PL \mathcal{C} -torsors.

November 2: Formulations of characteristic classes of flat bundles by Borel and Bott.

November 3: Homotopy axiom: $K_*(k) \rightarrow K_*(k[z])$.

November 8: K for vector bundles and coherent sheaves.

November 10: Good $k[z]$ -submodules.

November 12, 14: Homotopy axiom: $K_*^{\text{coh}}(A) \xrightarrow{\sim} K_*^{\text{coh}}(A[z])$ for noetherian rings.

November 15: Substitute for Tit's building.

November 17,19: More on K -systems.

November 22: Contractibility of the category with objects (P, u) where M is a fixed A -module and $u : P \rightarrow M$ surjective.

November 23: Filtrations of M .

November 28,29: Resolution problem.

November 29: Bisimplicial gadgets. Extending definition of class $\rho_E \in [X, BGL(A)]$ to sheaves.

1971-14

December 4: Chern classes of a flat bundle are torsion. Bott's theorem. Alternative approach using projective geometry.

December 5: Odd classes. Construction of refined Chern classes.

December 12: Odd classes in the p -adic case.

December 16: Simplicial complex associated to the p -subgroups of a finite group. Odd classes for families of representations.

December 24: Summary 'Functional Equation for ζ ': I complete non-singular curves; II number fields.

Contents 1972

1972-1

January 2: On $GL_n(K)$ where K is a local field of characteristic p . Compute $H^*(GL_n(K), \mathbb{F}_\ell)$, where $\ell \neq p$.

January 4: Continuous group actions, associated nerves and cochain complexes.

January 23: Wu formulae for $Sq^i(w_j)$. Notes for paper on K -theory.

1972-2

February 4: K -theory of bounded complexes in an exact category.

February 16: A category having the homotopy type $BGL(k)^+$.

February 18, 19, 20: Passage to the limit in the above category.

February 24, 25: Attempt at finding a 2-dimensional class associated to diffeomorphisms of compact support of \mathbb{R} (analogous to the 3-dimensional class belonging to a codimensional foliation).

February 25: Stable splitting of exact sequences of representations.

February 26: Another approach to Bott's formulae for Chern classes in terms of transition functions.

February 27: Closed forms on Segal's space. Explanation of motivation for seeking a central extension by \mathbb{R} of the group of diffeomorphisms of compact support of \mathbb{R} .

1972-3

March 1: Infinitesimal form of Čech cocycle.

March 2: Lie algebra extension defined by Čech cocycle. Review of Kostant's theory of a Lie algebra acting on a symplectic manifold.

March 3: On $SL_2(\mathbb{R})$. Ideas for future work. The Milnor model for BG .

March 4: Group completion theorem.

March 5: Integrating classifying topoi. Homotopy type of $\text{Nerv}(M \times M/\Delta M)$ in the case of K -theory.

March 6: Mumford's conjecture again. Idea for the proof that the category $\mathcal{J}(R)$ of pairs (V^+, V^-) with diagonal action is $BGL(R)^+$.

March 9: Eilenberg-MacLane cohomology.

March 11: Category $\mathcal{J}(J)$ where J is a space. Lang's theorem.

March 12: Stable Splitting theorem.

March 13: A cofibred category over \mathcal{J}

March 15: Review of power operations and the symmetric group. A model for $BGL(R)^+$.

March 17: $\text{Aut}(V)$ and the building $X(V)$ for a finite dimensional vector space over k .

March 19: Equivalence of categories \mathcal{J} and \mathcal{J}_o .

March 20: Buildings.

March 22: K -theory of $k[z]$ and of the projective line \mathbb{P} over k . Proving $K_*(k) \xrightarrow{\sim} K_*(k[z])$.

March 23: $H_G^*(X, \mathbb{F}_p) \xleftarrow{\sim} H_G^*(\text{pt}, \mathbb{F}_p)$ where k is a field of characteristic p , X is the building of proper subspaces of a finite dimensional vector spaces of V over k and $G = \text{Aut}(V)$.

March 24: Buildings.

March 25,26: To understand the mod p cohomology of $GL_2(K)$ where K is a local field of characteristic 0 and residue field of characteristic p .

March 29: Compactification of the building.

March 31: Why the last vertex function $\text{Nerv}(\mathcal{C} \rightarrow \mathcal{C})$ is a homotopy equivalence.

1972-4

April 2: Resolution problem.

April 15, 16: Education in statistical mechanics.

April 19: Exact sequences in K -theory.

1972-5

May 4: Conjectures on $K_*(F)$ where F is a function field.

May 5: Conjectures on $K_*(\overline{\mathbb{Q}})$.

May 6: Extending conjectures to \mathcal{O} , the ring of integers of a function field.

May 17: Compactification of $GL_n(\mathbb{R})/O_n$.

May 23: Vector bundles over a curve.

June 13, 14, 15: Fibration problem. Grothendieck's approach to the fundamental groupoid of a category.

June 17: Finite radicial maps of noetherian schemes and K -theory. Conjecture

$$\underline{\text{Hom}}(X, B) = B \underline{\text{Tors}}(X, B)$$

where \mathcal{C} is a category, $P \rightarrow B$ are \mathcal{C} -torsors such that fibres of $P \rightarrow \text{Ob}\mathcal{C}$ are contractible and X is a 'nice' space.

June 21: Cubes of morphisms with homotopy-cartesian sides.

June 23: Homotopy equivalence of fibres.

June 25: p -adic skew fields.

June 29: Homotopy theory of categories.

1972-6

July 7: Descent problem for a Galois extension $F \subset E$ with Galois group π .

July 9: To understand the ring $R = k[F]$ where k is an algebraically closed field of characteristic p and F is an indeterminate such that $Fx = x^q F$ for all $x \in k$.

July 14: Homotopy of categories again.

July 15: Remarks on Dold's paper on partitions of unity of fibrations (Annals 1963).

July 17: On interpreting $B\mathcal{S}d(I)$ as a subdivision of $B(I)$ where I an ordered set and $\mathcal{S}d(I)$ the ordered set of layers of I , .

July 18: Relations between Quillen's work on categories and Kan's E_X^∞ theory.

July 19: On $K_*(\mathbb{Z})$.

July 21: Remark on $\mathcal{C} \rightarrow \mathcal{C}'$, cofibred where each fibre is connected.

July 23, 25: On $\text{holim}_{\rightarrow}$. Homotopy of categories. Letter to John Milnor?.

July 31: On Stability. Finite generation of $K_*(A)$ where A is the ring of S -integers for a finite set S of places.

1972-7

August 1: Categories of projective modules over a Dedekind ring.

August 2, 4: Review of Borel-Serre results for $SL_2(\mathbb{Q})$. Homotopy inverse limits and how they relate to the descent problem in algebraic K -theory.

August 7: Finiteness for simplicial sets.

August 8: Category of simplicial G -sets.

August 9: Postnikov systems. KR-theory on the category of G -spaces.

August 10: Subdivision of a groupoid.

August 12: Barycentric subdivision of simplicial sets.

August 13: A simplicial set is replaceable by a simplicial complex.

August 23: Mumford's conjecture.

August 26: When \mathcal{C} is a category and $\text{Sub}(\mathcal{C})$ is a category with objects $X \rightarrow Y$ and morphisms from $(X' \rightarrow Y) \rightarrow (X \rightarrow Y)$ being diagrams

$$\begin{array}{ccc} X' & \leftarrow & X \\ \downarrow & & \downarrow \\ Y' & \rightarrow & Y \end{array},$$

what is $\text{Sub}(\text{Sub}(X))$?

August 27: Compactifying X/Γ . Category of p -subgroups of G . Parabolic buildings. Rank 2 bundles. Stable splitting theorem.

August 31: On $H_i(\text{Aut} M, \mathbb{Z}[\frac{1}{p}])$ where C is a projective non-singular curve over finite $k = H^0(C, \mathcal{O}_C)$ of characteristic p , $\Lambda = H^0(C \setminus \infty, \mathcal{O}_C)$ and M a projective Λ module of rank r .

1972-7

September 1: On $H^*(\text{Aut} M, \text{St}(M))$ where M is a finitely generated projective module over a Dedekind domain A and $\text{St}(M)$ is the Steinberg module of vector space $K \otimes_A M$ for K the fraction field of A .

September 7: Seattle conference. Acyclic maps, $K_*(A) = \pi_i(BGL(A)^+)$, $i \geq 1$. K -theory of symmetric groups = stable homotopy theory. Relation with cohomology theories derived from permutative categories, Review of ideas.

September 8: sflds. K -theory calculations.

September 10: Basic problems. Moore's theorem generalized.

September 17: Category $B(\coprod_k (X^k)_{\Sigma_k})$. Goals for the theory of an n -fold loop space.

September 19: To understand $K_*(A[\epsilon])$ where $\epsilon^2 = 0$, ϵ commutes with A , A is a commutative ring.

September 21: Why is $B\left(\coprod_{k \geq 0} X^k\right) = S(X_+)$?

September 22: Can we define an A -module structure on $\text{Ker}\{F(A[\epsilon]) \rightarrow F(A)\}$ where F is a functor from commutative rings to Ab ?

September 26: Cocartesian squares (up to homotopy) in the category of projective A modules of rank $\leq r$.

September 28: Model for $\Omega^\infty S^\infty$. Segal's simplicial space

$$\cdots \coprod_{mn} U_{mn} \begin{array}{c} \xrightarrow{\quad} \\ \xrightarrow{\quad} \end{array} \coprod_n U_n \rightrightarrows \text{pt.}$$

September 30: Spherical fibrations

1972-9

October 1,3: Segal's proof of the group completion theorem.

October 5: BG -theory - spherical fibrations. Difference for a topological monoid.

October 6: Segal's exploding spaces.

October 7: Products in exact sequence K -theory.

October 8,9, 10, 14, 15: New proof of group completion theorem.

October 15: Quasi-fibrations for simplicial sets. Problem: To prove directly that $BGL(k)^+$ is the fixed point set of Frobenius on $BGL(\overline{k})^+$.

October 17: Conjecture: If X is compact and \mathcal{C} is a small category then $\pi_0(\underline{\text{Tors}}(X, \mathcal{C})) \cong [X, B\mathcal{C}]$. Homology with coefficients in the Steinberg representation.

October 23: Show $B(\coprod U_n) \sim S^1 \times BU$.

October 24: Group completion theorem.

October 26, 27: Polynomial maps $f : V \rightarrow W$ which are additive for V, W finite dimensional vector spaces over an algebraically closed field of characteristic p .

October 27: Cobordism and K -theory.

October 28: Basic geometric facts about homotopy equivalences.

October 29: Lang's problem.

1972-10

November 1: To show that $Q(\mathcal{F}_A \rightarrow Q(\mathcal{P}_A))$ is essentially a covering in $K_0(A)/\mathbb{Z}$.

November 5: May's theorem.

November 7: Letter to Peter May setting out a new proof of group completion theorem. Classifying space of a simplicial monoid.

November 23: Periodicity calculations and Adams' operations.

1972-11

Notes on the 'bordism question'.

December 1: First proof of resolution problem (first page missing).

December 17: Exact categories.

December 21: Consequences in K -theory of the homotopy theorem.

December 23: Gersten's theorem and coherence.

1972-mixed papers and notes

Includes Tate's letter about the Lichenbaum conjecture; Chern classes associated to linear group actions on a finite dimensional vector space and a letter to Bott on characteristic classes associated to a foliation.

Contents 1973

1973-1

January 3: Gersten's conjecture: If A is a discrete valuation ring with residue field k , then the transfer map $K_*(k) \rightarrow K_*(A)$ is zero.

January 6: Exact categories with a resolving full exact subcategory.

January 10: Transfer for quasi-projective varieties.

January 11: An example of an additive category in which the Krull-Schmidt theorem holds.

January 13: Transfer.

January 19: On categories \mathcal{C} , $D(\mathcal{C})$ and $D_{lc}(\mathcal{C})$.

January 24: Groupoid \mathcal{S} with $+$ and \mathcal{S}/\mathcal{S} translation category.

January 27: Flat morphisms of noetherian rings.

January 28: Example connected to Gersten's conjecture.

January 29: Example of an irreducible polynomial.

February 1: Mumford's proof that $i^*i_*(y) = e(\nu)y$.

February 8, 10: Chow ring.

February 11: Spectral sequence $E_2^{pq} = H^p(X, \mathcal{K}_{-q}) \Rightarrow \mathcal{K}_{-p-q}(X)$.

February 13: History of paper "On the cohomology and K -theory of the general linear group over a finite field", and problems avoided.

February 1973: Exact sequence of closed subscheme.

1973-2

March 1973: Useful ideas not incorporated in 'Higher Algebraic K -theory'.

March 8: On a regular scheme whose local rings satisfy Gersten's conjecture.

March 10: Projective line and Lang's problem.

March 11, 13: Curves.

March 15: Education on curves. Elliptic curves.

March 73: Serre's theorem on non-vanishing sections.

March 22: On the homotopy type of small categories.

March 24: Category \mathcal{D} of pairs of finite dimensional vector spaces over a field k .

March 25: To understand stability.

March 27: On $GL_2(K)$, K a field.

March 28, 30: Stability continued. Inverting.

1973-3

April 5,7, 8: More stability.

April 9, 10, 22: The Q category in classical K -theory.

April 14: $GL_2(\mathbb{C})$.

April 16: $GL_n(k[t])$ and vector bundles over curves.

April 18 Localization and Splitting theorem.

April 19: ζ functions.

April 20: Cohomology calculations.

April 24: Localization via Serre's method.

April 28: For a given ring A , explicit construction of a space representing K -theory of A of a given weight.

April 30: Becker-Gottlieb proof of Adam's conjecture.

1973-4

May 6: Stable bundles on a Riemann surface. Mumford's method for constructing a moduli space. Mumford-Sheshadri example of an exact category. Vector bundles on an elliptic curve (Atiyah's paper).

May 10: Possible proof of $K_q(A[t]) = K_q(A) \oplus \text{Nil}_{q-1}(A)$ (Quillen comments "no good").

June 6; $p^n A = 0 \Rightarrow K_n(A)[p^{-1}] \xrightarrow{\sim} K_n(A[t])[p^{-1}]$.

June 16: B regular coherent implies $K_0(B) \xrightarrow{\sim} K(B[t])$.

July 11, 30: Monoidal categories. Action preserving functors.

1973-5

October 1973 Construction of a spectrum associated to an exact category. K -theory of an exact category.

October 8, 9: Notes on Wagner's paper. Category interpretation.

October 15: Grassmannian - Schubert geometry.

October 16: Serre's theory on stability.

October 22: λ -operations. Construction of products and λ -operations in K -theory for a scheme.

October 24: On buildings (after a conversation with Serre).

October 25: Some lemmas about classifying spaces.

1973-6

November 5: Double mapping cylinder. Classifying spaces of a topological monoid.

*(November 6, 1974 - January 8, 1974): Serre's course on the cohomology of discrete groups.

*January 8 1974: Serre's lectures.

November 7: Serre's seminar on Manin's work.

November: Letter from Dennis Sullivan and reply about a problem of Bob Williams.

1973-7

December 7, 8, 9, 10: Lusztig's work. Lusztig's complex $L(V)$.

December 11: Spherical simplicial complexes.

December 13: Theorem: If G_S is the group of permutations of an infinite set then $\tilde{H}_0(G_S) = 0$.

December 15: The group of automorphisms of a vector space of the form $1 + \theta$ where θ has finite rank.

December 18: Grassmannians.

1973-8

December 19, 20: More Grassmannians.

December 20: Lusztig - Kervaire approach to buildings.

December 21, 22, 24, 27: Stability.

December 29: Poincaré duality and Zeeman spectral sequence.

December 30: Grassmannians. Removing simplices in a quasi-spherical complex.

December 31: Euler characteristics and Stiefel-Whitney homology class.

1973-9

Rough outline of Part I: (1) Homotopy type of categories, (2) S^{-1} construction.

1973-10 Continuing part I. Fibration $\mathcal{S}^{-1}\mathcal{S} \rightarrow \mathcal{S}^{-1}\chi \rightarrow \langle \mathcal{S}, \chi \rangle$. Relation with Bass-Milnor approach. Splitting theorem. (3) Splitting of exact sequence. Results of second part: (1) Comparison theorem, (2) Resolution, (3) Localization, (4) Fundamental Theorem, (5) Applications of fundamental theorem.

1973-11 Comparison theorem. Notes produced in preparation of ‘Higher K -theory for Categories with Exact Sequences.’

Contents 1974

1974-1

January 1974: The stability problem for $GL(k)$, $k = \mathbb{F}_q$.

January 3: Stability proofs.

January 4: 2-planes and bouquets of spheres.

January 6, 7: More stability.

January 7: Lusztig's $D(V)$.

January 9: G -spaces. Steifel-Whitney classes of Euler spaces.

January 10: Stability for a field.

January 11: 2-torsion peculiarities in stability.

January 12: Computation of Seiberger homology.

January 14: Stability proof on the 'direct-sum' situation.

January 18: Stability proof for $H_*(GL_n)$ using the unimodular group. Categories obtained by gluing procedure

$$\begin{array}{ccc} (X, G) & \hookrightarrow & (CX, G) \\ \downarrow & & \downarrow \\ \mathcal{C}' & \hookrightarrow & \mathcal{C} \end{array}$$

where X is a G -space.

1974-2

January 19: The partially ordered set T_X of non-empty totally ordered finite subsets of an infinite set X , and applications to stability for $H_*(GL_*)$.

January 21, 22: Stability for a field k .

January 23: $PGL_3(k)$. Matsamuko result.

January 24: Classifying space.

January 27: Stability.

January 30: Crude stability.

January 31: Weak stability for a Dedekind domain.

1974-3

February 5: Localization.

February 7, 10, 11: Nagao theorem.

February 11: $GL_2(\mathbb{Z})$ and $GL_3(\mathbb{Z})$,

February 13: Sylvester's theorem.

February 17: Waldhausen.

March 3: Waldhausen.

March 6: Localization.

March 7: Vector bundles over an elliptic curve C .

March 8: Sylvester's theorem and C/Γ .

March 9: Elliptic curves.

March 10: Discrete valuation rings.

March 17: Waldhausen continued.

1974-4

March 26: Elliptic curves.

March 27: Curves.

March 30: Siegel formula.

April 1, 2: Siegel formula.

1974-5

April 3: Euler characterisitic $\chi(\Gamma, I)$ where M is a rank 2 bundle over $C \setminus \infty$, X the tree of extensions of M to C , $\Gamma = \text{Aut}(M)$, and $I(M) = H_1(X, X_{\text{inst}})$.

April 6: Euler characterisitic formula - p -adic analogue. Remarks on $\chi(\Gamma)$. Upper half plane and quadratic forms.

April 7: More Euler characterisitic.

April 8: Rank 3 bundles.

April 9: Link of a point in a Tits building of lattices.

April 10, 11: Vector bundles over a curve C considered as a lattice in a vector space over the function field of C .

April 12, 13: Counting stable bundles. Net conjecture.

April 15: Net conjecture for $r = 3$, $d_\infty = 1$.

1974-6

April 18: Net conjecture continued.

April 19: Question: Let x be a semi-stable point in X the Tits building (April 3). Does $\text{Aut}(x)(\text{mod scalars})$ act faithfully on the link of x ?

April 23, 24, 25: Localization.

April 26: On ζ for an elliptic curve.

April ?, 27, 28: Lang problem again.

April 29: On the homotopy equivalence $BGL(\mathbb{F}_p[F^{-1}, F])^+ \rightarrow Q(\mathcal{P}(\mathbb{F}_p))$.

1974-7

May 1: Lang problem.

May 7: Grassmannian p planes in $p + q$ space.

May 9, 11, 13: Stratification of $\text{Hom}(E, F)$ where E and F are two vector bundles of the same rank.

May 15: Two models for BU .

May 17: Interpretation of Q category for topological K -theory.

May 19: Stratifications.

May 20: K -theory for quadratic modules.

1974-8

May 28: Karoubi's periodicity theorem with Andrew's suggestion and ideas of Graeme Segal.

June 1, 3, 4, 10, 11: K -homology.

1974-9

Associated notes including material on families of unitary representations, signature formula, double mapping cylinder.

August 8: Example from Dold-Thom theory of quasi-fibration space A_X where A is a Banach algebra and X is a space.

August 9, 10: Coverings.

1974-9

August 13: "Space" of "chains" on a compact space T with coefficients in an exact category \mathcal{P} . Finite generation of K -groups in function field case.

August 15: Continuing with K -theory.

1974-11

September 2: Mumford's view of GL_n at ∞ .

September 5: Grassmannians.

September 7: Kuiper's theorem.

September 8: Constructing $k(X; T)$ from vector bundles over space X with T decompositions for a finite complex T .

September 14: Realizing finite sets.

September 19: Grassmannian.

1974-12

October 4: On SA , the suspension of a field A .

October 6: Conjecture: The ordered set $L_q(V)$ of layers of dimension q in an infinite dimensional vector space V is a classifying space for GL_q .

October 7, 8: Generalizing to the Q category the Kervaire-Lusztig contraction argument. Volodin approach to grassmannians.

October 11: Two varying layers in a vector space.

1974-13

October 11, 13, 15: Schubert cells.

October 20: Schubert cells and flag manifolds.

October 24: Relating grassmannians and Fredholm operators.

1974-14

Karoubi's theorem. $K_1(SA) = K_0(A)$.

October 25: Karoubi's periodicity theorem. Simpler proof.

October 29: Periodicity modulo M .

November 2: Periodicity theorem continued. Factoring $S^1 \rightarrow GL_n(\mathbb{C})$. Scattering version.

November 7, 10: ΩGL_n .

1974-15

November 20, 21: On $\mathcal{L} = GL_n(\mathbb{C}[z, z^{-1}])' / GL_n(\mathbb{C}[z])'$. (Note that 16 pages are missing.)

November 22: problem: To find the universal topological group G equipped with a continuous map $\varphi\{A \in GL_n : A^m = 1\} \rightarrow G$ such that $\varphi(AB) = \varphi(A)\varphi(B)$ if A and B commute.

November 24: Review.

1974-16

Notes on the $+$ construction.

1974-17

K -theory of a ring.

1974-18

December 15: Bruhat decomposition for GL_n .

December 17, 19: Homotopy type of the poset of Schubert cells.

December 22: Cohomology associated to Schubert cells.

December 23: Flags and Schubert cells.

1974-19

Summary of research Dec 1974 - January 1975 concerning Schubert cells and buildings.

December 28: Schubert cells.

December 31: Cohomology associated to Schubert cells.

1974-20

Notes included with 1974 material. Much of this material is related to 1974-10.

Contents 1975

1975-1

1975 Preliminary notes on building and loops for a compact, 1-connected symmetric space.
The loop space of a symmetric space. Buildings and the loop space of a Lie group. [1975-2](#)

Notes on building and symmetric spaces, Part I

1975-3

Notes on buildings and symmetric spaces, Part II.

1975-4

Notes on buildings and symmetric spaces, Part III.

1975-5

Notes on buildings and symmetric spaces, Part IV,

1975-6

Notes on buildings and symmetric spaces, Part V.

1975-7

January 1-10: On Schubert cells.

1975-8

January 12-19: On Schubert cells.

January 20: Review.

1975-9

January 23: The nerve of the covering of $\mathbb{P}(V)$ given by complements of hyperplanes.

January 25: A modification of the the category BGL .

February 3: A "space" of rank 2 bundles over a non-singular curve.

February 9: Conventions concerning lattices, Laurent polynomials, etc:

$\mathcal{G} = GL_n(\mathbb{C}[z, z^{-1}])$, $\mathcal{G}' = \{\alpha \in \mathcal{G} : \alpha(1) = 1\}$, $\mathcal{U} = \{\alpha \in \mathcal{G} : |z| = 1 \Rightarrow \alpha(z) \in U_n\}$,
 $\mathcal{U}' = \mathcal{U} \cap \mathcal{G}'$.

February 14: Cell decomposition of a flag manifold. Cell decomposition in space of lattices.

1975-10

February 14: G -spaces.

February 21: Stability for Σ_n .

February 22: Category C_n of non-empty finite sets of cardinality $\leq n$ and injective maps.
Review of Volodin-Wagner construction.

February 24: Modifying $B\Sigma_n$.

February 26: Examples of U_n -manifolds.

March 1: Review of a stratification of the Grassmannian.

March 14: The group SU .

March 15: Slope polynomials.

March 16: K -theory of a curve.

March 19: Resolution theorem. Deformation of a rank 2 bundle.

March 23: More on SU ,

March 31: More on Schubert cells.

1975-11

May 1975 The building for SL_n over a field with a valuation.

May 1975 Scattering theory.

1975-12

May 1975 Lattices and scattering theory.

June 6: On symmetric spaces.

1975-13

June 9, 12: Roots for a symmetric space.

June 12-15: Buildings continued.

1975-14

June 17 - 227: Morse theory and Lie groups. Grassmannians as symmetric spaces. The relation between Quillen and Bott-Samuelson approaches to ΩX . Bruhat decomposition in symmetric spaces.

July 1: Bruhat decomposition for a spherical building.

1975-15

July 3: Stratifying the action of a compact connected Lie group K on its Lie algebra. More on good K spaces. Self normalizing subgroups.

July 3-11: Buildings.

1975-16

July 11: Wiener factorization. Garland-Raghunathan proof. Direct proof that $\mathcal{U}' \rightarrow \Omega U_n$ is a weak homotopy equivalence.

July 24: Closures of Schubert cells.

1975-17

August 11, 18: Compactifying a symmetric space.

August 14: Representation of U_m .

August 15: Deligne's λ -operations.

August 21: Relation between the two vector fields on $K\eta$.

September 1: Morse theory.

1975-18

September 7-19: Discrete series and cusp forms.

1975-19

September 20: Green algebras for Σ_n and $GL_n(\mathbb{F}_q)$.

September 21: Modular representation of $GL_n(\mathbb{F}_q)$. Mackey formula.

September 23-26 The Hopf algebra structures on $\oplus H^*(GL_n(\mathbb{F}_q))$ and $\oplus R(\Sigma_n)^\vee$.

1975-20

October 1: To define a category of chains on a simplicial complex with coefficients in an exact category.

October 3: A discussion of the building of the complexification G of a compact Lie group K when the building is identified with the unit sphere in $\text{Lie}(K)$.

October 3, 4: Cerf's paper on pseudo-isotopy.

October 5: Wagner's construction.

October 10: Application of the h-cobordism theorem to studying fibre bundles where each fibre is a contractible n -manifold. Old idea to find a geometric proof that K -theory of finite sets is stable homotopy.

October 12: Problem: Let $M \rightarrow X$ be a differentiable fibre bundles and Y the zero-submanifold of a generic section of the tangent bundle along the fibres of M/X . Describe Y/X as a structure over a stratification of X .

October 14: Let E, F be vector spaces over a field k and determine the orbits in $\text{Gr}_r(E \oplus F)$ for the group $\text{Aut}(E) \times \text{Aut}(F)$.

1975-21

October 17: Grayson's discovery.

October 18-24: On MacPherson's construction.

October 24-27: Sheaves over a finite poset.

October 29-31: The poset attached to a vector space.

November 1: In a vector space the category of distributive lattices is equivalent to the dual of the category of posets.

1975-22

November 3: (Pages 1-7 missing) Let V be a vector space over a field K of countable infinite dimension. Study the poset X of subspaces W of V such that $\dim W = \dim V/W = \infty$ with $W_1 < W_2$ iff $W_1 \subset W_2$ and $\dim W_2/W_1 = \infty$. Question: Is X contractible.

November 5-12: A stability theorem.

November 17, 19: On the simplicial complex $Y_2(S)$ associated to a finite set S where the points v are 2 point subsets and $\{v_0, \dots, v_p\}$ is a simplex if the v_i 's are disjoint.

November 19: Group completion.

November 20: What is the homotopy type of a category \mathcal{C} of triples (E, k, F) where E and F are sets, k an integer and a map $(E, k, F) \leftarrow (E', k', F')$ consists of a pair of isomorphisms $E \xleftarrow{\sim} E' \oplus A^\mu$, $F \xrightarrow{\sim} A^\nu \oplus F'$ such that $\mu + k' + \nu = k$.

November 23: To construct a category realizing the simplicial group associated to a free simplicial monoid.

November 30: How topological and algebraic K -theory differ. On relation to work of Volodin and Wagner, and of Loday.

1975-23

December 5-13: The K -theory of coherent sheaves on an algebraic variety.

December 14-23: Brown-Gersten spectral sequence.

December 24: Can the Bass relative group $K(\Phi)$ be made into a higher K -theory? Bass K group.

December 26: Bass K groups for a finite field. 'Makin' K -theories.

December 28-31: Determine the space of torsion sheaves on \mathbb{P}^1 .

Contents 1976

[1976-1](#), [1976-2](#), [1976-3](#)

The notes in these files are concerned with aspects of vector bundles on \mathbb{P}_A^1 and \mathbb{A}_A^1 .

[1976-6](#), [1976-7](#), [1976-8](#), [1976-9](#)

The notes in these files were written in preparation for the article 'Homotopy Properties of the Poset of Nontrivial p -Subgroups of a Group.

[Note by Quillen](#)

A comment on what Quillen regards as his best work.

Contents 1977

1977-1

De Branges spaces

1977-2

January 5- 28: Statistical Mechanics

1977-3

January 29: Ising inequalities and related calculations.

February 1: Classifying semi-modular functions on $L \times \{0 < 1\}$ where L is a finite distributive lattice.

1977-4

February 4: More on Ising inequalities. Review of knowledge of the ζ -function.

February 5: Bernoulli trials. Γ -function.

February 6, 8, 9: Lee-Yang theorem (Asano proof).

1977-5

February 10: Calculations connected with $Z(s) = \zeta(s)\pi^{s/2}\Gamma(s/2)$.

February 11: Solution of heat equation $\frac{\partial u}{\partial t} = a \frac{\partial^2 u}{\partial^2 t}$ on \mathbb{R} with $a > 0$ and connection with $Z(s)$.

February 13, 14: Functions equal to their Fourier transforms and connections with $\zeta(s)$.

February 17: Identity $\sum e^{-\pi n^2 t} = \frac{1}{\sqrt{t}} \sum e^{\pi n^2 t^{-1}}$.

February 18: Back to Lee-Yang polynomials.

1977-6

February 19: Monomer-dimer problem (after Hohlman-Lieb).

February 23, 24: Ising polynomial.

February 25: Lee-Yang article, Phys. Rev. 87, 404-410.

February 26: Review of Euclid's algorithm and continued fractions. Ising model again.

February 27: Unimodular matrices which preserve the unit disc.

February 28: Chain of connected oscillators and Jacobi matrix associated to the potential energy. Jacobi matrices, truncations, continued fractions and associated difference equations. Relation between orthogonal polynomials and operators on Hilbert spaces. Measure

associated to a Hilbert space with an operator A and cyclic vector v_0 : $1 \mapsto v_0$, $x \mapsto Av_0$ and such that $\int p(x)d\mu = (p(A)v_0, v_0)$. Recurrence relation and Jacobi matrices associated to orthogonal polynomials. Continued fractions and moments $c_n = \int x^n d\mu$.

1977-7

March 1: Periodic Jacobi matrices. Orthogonal polynomials on $|z| = 1$.

March 5: Doubly infinite Jacobi matrices. Spectrum of a Jacobi matrix.

1977-8

March 5: Relation between a double infinite Jacobi matrix and associated one-sided Jacobi matrix. Isospectral deformation.

March 6, 7: Unimodular transformations on the upper half plane and on the unit disc. Linear Ising chain with periodic conditions and partition function. Heilmann-Lieb dimer limit of the partition function.

March 9: Classify all holomorphic $u \mapsto A(u)$ with $\text{tr} A = 0$, $A(u)$ real when u real, and several other conditions described on March 7.

1977-9

March 10: Second order differential equations associated to discrete strings.

March 11: The equation $\frac{d^2 y}{dx^2} + \lambda y = 0$. Bessel's functions.

March 12: Inverse Fourier transform of $\cos \sqrt{u^2 - a^2}$.

March 14: L -function for $\mathbb{Z}[i]$.

1977-10

March 15: Doubly infinite Jacobi matrices of period n .

March 16: Recursion formulas for $(J - \lambda)y = 0$.

March 19: Summary of some material from Lax's paper 'Almost periodic behaviour of non-linear waves.' *Advances in Math.* 16 (1975), 368-379.

March 20: Given an infinite Jacobi matrix, determine matrices giving rise to an isospectral deformation.

March 21, 23: Flow on spaces of J -matrices $L = aT + b + T^{-1}a$ where T is the shift operator. Potential scattering $L = \Delta + q$ where q has compact support and Green's function/resolvent G .

March 24: Correspondence between J -matrices and measure on \mathbb{R} .

1977-11

March 25: Approximating a Sturm-Liouville DE with a Jacobi matrix.

March 29: Inverse scattering à la Kac.

March 30: Scattering for a J-matrix which agrees with $L = \frac{1}{2}T + \frac{1}{2}T^{-1}$ outside a compact region.

March 31: Scattering for $L = aT + b + T^{-1}a$ where $a_n = \frac{1}{2}$ and $b_n = 0$ for $|n|$ large.

April 1: For $S \in SL_2(\mathbb{C})$, calculate the image of $\mathbb{P}_1(\mathbb{R})$ in $\mathbb{P}_1(\mathbb{C})$ under S^{-1} and apply to the Sturm-Liouville system $\frac{d}{dt} \begin{pmatrix} u \\ u' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ q - \lambda & 0 \end{pmatrix} \begin{pmatrix} u \\ u' \end{pmatrix}$ and Ising model.

1977-12

Eigenvalue distribution of Bessel function $K_S(2\pi)$ using Bohr-Sommerfeld rules.

April 2: Study $\frac{dX}{dx} = (A_0 + \lambda) \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} X$ with A_0 a real symmetric matrix. Work out the relation between the solution matrix $S(\lambda)$, the spectral measure and the Green's function.

April 4, 5: On Bessel function $K_s(c) = \int_0^\infty e^{-c/2(t+t^{-1})} t^s \frac{dt}{t}$.

April 9, 10: Eigenvalue distribution for $\frac{d^2u}{dx^2} + pu = \lambda qu$, p real.

1977-13

April 11, 13, 15, 17, 18: The spectral function of a Sturm-Liouville problem. Hörmander's method.

1977-14

April 20, 22: Transform $-\frac{d^2\psi}{dx^2} + q\psi = \lambda^2\psi$ into the wave equation by taking the Fourier transform with respect to λ .

April 23: Example: $\frac{\partial^2 u}{\partial t^2} = \left(r \frac{\partial}{\partial r}\right)^2 u - r^2 u$.

April 24, 25, 26: On $\frac{1}{i} \begin{pmatrix} \frac{d}{dx} & -p \\ p & -\frac{d}{dx} \end{pmatrix} \phi = \lambda \phi$ and $\frac{1}{i} \frac{\partial u}{\partial t} + \begin{pmatrix} \frac{\partial}{\partial x} & -p \\ p & -\frac{\partial}{\partial x} \end{pmatrix} u = 0$. Hörmander's analysis of $\frac{1}{i} \begin{pmatrix} \frac{d}{dx} & -p \\ p & -\frac{d}{dx} \end{pmatrix} \phi = A\phi$.

1977-15

April 27: $\frac{dX}{dt} = AX$ where $A = A_0(t) + A_1(t)\lambda$ and Ising model limits.

April 28: Classical motion associated to $\left(\frac{dx}{dt}\right)^2 + e^{2x} = \lambda^2$.

April 29, 30: Schrödinger's equation $-\frac{\hbar^2}{2m} \frac{d^2\phi}{dx^2} + V(x)\phi = E\phi$. Heat equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$.

May 1, 2, 4: On $\theta(x, t) = \sum_{n \in \mathbb{Z}} e^{\pi n^2 t} e^{2\pi i n x}$. Asymptotic expansion of $S(x, \lambda)$ where $u = e^{iS(x, \lambda)}$ is a solution of $-\frac{d^2 u}{dx^2} + qu = \lambda u$.

May 11: Extension to work of April 1.

1977-16

May 13: $-\frac{d^2 u}{dx^2} + x^2 u = \lambda u$, Titchmarsh approach.

May 14: Review Γ -function.

May 14: $-\frac{d^2 u}{dx^2} + (\lambda - q)u = 0$ again.

May 16: Fourier transform of the characteristic function of the set $[-W, W]$.

May 18, 19, 20, 21: $\frac{d}{dx}u = \begin{pmatrix} i\lambda & \bar{p} \\ p & -i\lambda \end{pmatrix} u$ and associated Ricatti equation.

May 22, 23: Analysis of $u'' + (\lambda^2 - q)u = 0$

May 25: Comment on Weyl meromorphic equation.

May 26: Strings.

1977-17

May 27: Spectral measure.

May 28: Whittaker's function $W_{k,m}$, a solution to the confluent hyper-geometric differential equation.

May 29: Distribution of eigenvalues.

May 30: Connection between solutions of $\frac{d}{dx}u = \begin{pmatrix} i\lambda & x \\ x & -i\lambda \end{pmatrix} u$ and the Γ -function. Properties of ζ -function.

June 1, 2, 3: Study of $H(x, y, s) = \sum_{n=0}^{\infty} (x+n)^{-s} e^{2\pi i n y}$ and $G(x, y, s) = \sum_{n \in \mathbb{Z}} (x+n)^{-s} e^{2\pi i n y}$

1977-18

June 3: Continue study of $G(x, y, s)$. More on the θ -transformation formula.

June 4: Continuing analysis of May 30.

June 5: Duplication formula for the Γ -function. Bessel functions. Hankel functions.

June 6: Wronskian of solutions of $\left(\frac{d^2}{dr^2} + \frac{1}{r} - 1 - \frac{s^2}{r^2}\right)u = 0$.

June 7: Generalizations of the Legendre duplication formula.

1977-19

June 10: Simple harmonic oscillator in 2-dimensions.

June 11: Bessel functions again. Polya's proof of the functional equation for $\theta(s)$.

June 12: Random walk on a cyclic group of order n .

June 14: Hartman's estimate of $N(\lambda)$ associated with $u'' + (\lambda - q)u = 0$.

June 15: Eigenvalue distribution for $\frac{du}{dx} = \begin{pmatrix} i\lambda & \bar{o} \\ p & -i\lambda \end{pmatrix} u$.

June 16: Return to analysis of $\left(\frac{d^2}{dx^2} + \frac{1}{r}\frac{d}{dx} - \frac{m^2}{r^2} - r^2\right)u = 0$.

June 17: Confluent hyper-geometric differential equation: $x\frac{d^2y}{dx^2} + (c-x)\frac{dy}{dx} - ay = 0$.

June 18: Hermite differential equation: $\left(\frac{d}{dx} - x\right)\left(\frac{d}{dx} + x\right)u = 2nu$.

June 19, 20: L-functions.

June 21: Lerch's formula. Lee-Yang polynomials.

1977-20

June 22: Functional equation for $H(x, y, s) = \sum_{n=0}^{\infty} e^{2\pi i n y} (x+n)^{-s}$. Dirichlet L -functions.

June 23: Dirichlet series $\sum_{n \geq 0} a_n n^s$ where a_n depends on n modulo p . Quadratic Gaussian sums $\theta(t) = \sum e^{-\pi n^2 t}$.

June 24, 26: Dirichlet series $L(s, \chi) = \sum_{n=1}^{\infty} \chi(n) n^{-s}$ where $\chi : \mathbb{Z}/p\mathbb{Z} \rightarrow \mathbb{C}$. Fourier transforms of Gaussian sums. Quadratic reciprocity.

1977-21

June 26, 27: Gauss sums.

June 28, 29: Hermite-Weber differential equation.

July 2: $\frac{d}{dx}(u) = \begin{pmatrix} i(\lambda + x) & \rho \\ \rho & -i(\lambda + x) \end{pmatrix}$, ρ real constant. [1977-22](#)

July 3: Difference equation analogues.

July 4: Review J -matrices. Transitions between equivalent 'gadgets'

July 5: Difference equation $y_{n+1} - 2zy_n + 2ny_{n-1} = 0$.

July 7, 8, 9: Zoo of examples of difference equations. Orthogonal polynomials.

July 11, 12, 13: Review the way $\text{Todd}(x) = \frac{x}{1-e^x}$ occurs in topology.

Discrete analogue of $\frac{du}{dx} = \begin{pmatrix} i(\lambda + b) & \rho \\ \rho & -i(\lambda + b) \end{pmatrix} u$. Recursion relations satisfied by hyper-geometric series.

1977-23

Hahn's q -difference equations. Example where L is a linear operator on polynomials decreasing degree by 1 and such that $L(xf) = f + (qx + w)L(f)$. $L(f) = \frac{f(qx) - f(x)}{(q-1)x}$.

July 16, 17: First order difference equation with rational coefficients.

July 18: Differential equations of the form

$$(c_1x^2 \frac{d^2}{dx^2} + c_2x \frac{d}{dx} + c_3)y = (c_4x^3 \frac{d^2}{dx^2} + c_5x^2 \frac{d}{dx} + c_6x)y$$

and related difference equation.

1977-24

July 19: Relation between q -difference equations and vector bundles on the elliptic curve $X = \mathbb{C}/\langle q \rangle$.

July 20: General facts about q -difference equations: $f(x) = A(x)f(qx)$.

July 21, 22: Difference equation:

$$c_1f(x) + c_2f(qx) + c_3f(q^2x) = x(c_4f(x) + c_5f(qx) + c_6f(q^2x))$$

1977-25

July 23: Mellin transform. Kummer equation. More on July 21 equation.

July 24: $\left(\frac{d}{dx} - x\right)\left(\frac{d}{dx} + x\right)u = 2su$ and its Wronskian. Comment on realising ζ -function as a Wronskian.

July 24, 25, 26: More on July 21 equation.

July 27: Eisenstein's continued fraction. Padé table.

July 28: Review connections between probability measure on \mathbb{R} , orthogonal functions, difference equations and continued fractions.

July 29: Symmetric two-sided Jacobi matrix describing the recurrence relation $\lambda y_n = a_n y_{n+1} + b_n y_n + a_{n-1} y_{n-1}$ and continued fractions formula.

July 31, August 1: Solution to modified Bessel's differential equation $\left[\left(r \frac{d}{dr} \right)^2 - r^2 - s^2 \right] u = 0$.
 q -analogue of Bessel's equation.

1977-26

August 2: More on $\lambda y_n = a_n y_{n+1} + b_n y_n + a_{n-1} y_{n-1}$.

August 3: When is the spectrum of a Sturm-Liouville operator $\frac{d}{dx} p \frac{du}{dx} + (\lambda x - q)u = 0$ discrete?

August 4: More on Jacobi matrices and continued fractions. Discrete strings.

August 5: Review scheme for relating $\zeta(s)$ to the system $\frac{du}{dx} = \begin{pmatrix} i\lambda & p \\ \lambda & -i\lambda \end{pmatrix} u$.

August 6; p real in August 5 system.

August 8: Γ -function and Mellin transform of $\frac{1}{1+t}$

August 10: Expansion of $\pi \cot(\pi t)$. Functional equation of $\zeta(s)$. Wish to fit ζ -function into *self-adjoint on the line* setup.

August 11: Zeta function of a curve. Riemann proof of the θ -transformation formula by contour integration and another simpler method using geometric series.

August 12: More on $\frac{du}{dx} = \begin{pmatrix} i\lambda & \bar{p} \\ p & -i\lambda \end{pmatrix} u$.

August 13 Review $\int_L \frac{e^{-\pi t x^2 + 2\pi i p x}}{e^{2\pi i x} - 1} dx$. Contour integrals $\int \frac{e^{ax^2} x^s}{e^{\pi i x} - e^{-\pi i x}} \frac{dx}{x}$.

August 14: on $f(y) = \int_0^\infty \frac{e^{-yt}}{t+a} t^s \frac{dt}{t}$.

1977-27

August 15: Return to $\frac{du}{dx} = \begin{pmatrix} i\lambda & \bar{p} \\ p & -i\lambda \end{pmatrix} u$ and connection with $\zeta(s)$.

August 16: To understand the theory of strings due to Krein, generalizing Stieltjes theory of continued fraction and the moment problem.

August 19, 21, 22: De Branges functions and solution to $\frac{du}{dx} = \begin{pmatrix} i\lambda & \bar{p} \\ p & -i\lambda \end{pmatrix} u$.

August 25, 26: On $\frac{d^2 u}{dx^2} + (\lambda - q)u = 0$ and $L = \frac{1}{i} \begin{pmatrix} \frac{d}{dx} & -\bar{p} \\ p & -\frac{d}{dx} \end{pmatrix}$.

August 27: De Branges function E and associated space $B(E)$.

1977-28

August 28: More on de Branges space $B(E)$.

August 29: $B(E)$ when E is a polynomial with roots in the lower half plane.

September 2, 3, 4, 7: Finite measures on \mathbb{R} and associated sequences of orthonormal polynomials. De Branges version of same material.

1977-29

September 14: Scattering theory, J -matrices with $a_i = \frac{1}{2}$, $b_i = 0$.

September 17: Stieltjes transform of a measure on \mathbb{R} and how to recover μ from f where
$$f(\lambda) = \int \frac{d\mu(\hat{\lambda})}{\lambda - \hat{\lambda}}.$$

September 18, 19: Continued fractions expansions of rational functions and associated J -matrices.

September 25, 26: Time-dependent scattering.

September 26, October 1-5: Comparing time-dependent scattering based on unitary groups e^{-itH} and e^{-itH_0} and that based on the Schrödinger equation.

1977-30

October 7-11: Radial Schrödinger equation. Hormander's approach to its spectrum.

October 15: Holomorphic functions on the unit disc. Herglotz's formula.

October 19:
$$\int \frac{d\mu(\lambda)}{\lambda - \mu} = \frac{1}{\lambda - b_1} \frac{a_1^2}{\lambda - b_2} \cdots \frac{a_{n-1}^2}{\lambda - b_n}.$$

October 21: Review scattering for the Dirac equation $\frac{du}{dx} = \begin{pmatrix} i\lambda & \bar{p} \\ p & -i\lambda \end{pmatrix} u$, $0 \leq x < \infty$, $u_1(0) = e^{i\alpha} u_2(0)$, p compact support and discrete analogue.

October 26: Special orthonormal bases for $L^2(S^1, d\nu)$.

October 29: Herglotz formulas for the unit disc and corresponding upper half plane results.

October 30, 31: More on scattering.

November 2, 4: Schur's analysis of function $f(z)$ holomorphic on $|z| < 1$ and satisfying $|f(z)| \leq 1$.

November 5, 6: Review discrete strings.

November 9: Paradox of indeterminate moments.

1977-31

November 11: Abstract scattering theory.

November 12: Scattering associated to a Diract-style system.

November 13: Positive definite functions on groups.

November 20, 25: Relating de Branges theory with second order systems.

November 27, 30: Converting a Sturm-Liouville system to string form. Scattering analysis.

1977-32

December 3,4: Summary of November 27 analysis.

December 5, 7, 9: Dirac system analysis with $p = \delta$, the delta function.

December 10: On $SU(1, 1) \subseteq SL_2(\mathbb{C})$.

December 11: The Lee-Yang polynomial belonging to as linear graph is essentially the charactristic polynomial of a unitary operator.

1977-33

December 14: Wave equation approach to $-\frac{\partial^2 U}{\partial x^2} + Vu = \lambda^2 u$.

December 16: Gelfand-Levitan approach.

December 18, 19, 20: Dirac equation again.

December 21: Fourier transform of a measure $d\mu(\lambda)$ on \mathbb{R} .

December 22: Fourier transform of a measure on S^1 .

December 24, 25, 26: Toeplitz form, i.e., orthonormal polynomials. Equivalent formulations.

December 28: On $(-\frac{d^2}{dx^2} + q)u = \lambda^2 u$ again.

1977-34

Notes on Marchenko equation.

December 29, 30: Marchenko equation.

Contents 1978

1978-1

January 1,2: De Branges ideas on analysis for the equations $Lu = P\frac{du}{dx} + Qu = \lambda u$, $P = \begin{pmatrix} \frac{1}{i} & 0 \\ 0 & i \end{pmatrix}$, and $Lu = (-u'') + qu = \lambda u$.

January 3,4: Lee-Yang theory.

1978-2

January 4,5: Review discrete wave equations.

January 6: On Dirac system where the p is a sum of δ -functions at the integers.

January 7: On a probability measure ν on S^1 and an associated orthonormal polynomial basis for $L^2(S^1, d\nu)$.

January 8: Review Bessel K -function and de Branges theorem.

January 9: A heuristic explanation for the completeness relation.

January 11: A J -matrix eigenvalue problem.

1978-3

January 12,13: A Dirac system on $0 \leq x < \infty$ with $\phi(0, \lambda) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

January 14: On $Lu = -u'' + qu = \lambda u$, $0 \leq x \leq \ell$.

January 16: De Branges approach to completeness. Scattering theory for Schrödinger's equation on \mathbb{R} .

1978-4

January 17,18: Electrical circuits.

January 19: Scattering for the Dirac system $\frac{du}{dx} = \begin{pmatrix} i(\lambda - a) & \bar{p} \\ p & -i(\lambda - a) \end{pmatrix} u$.

January 20,21: Power flow for a circuit.

January 22: On a linear mechanical system consisting of n particles.

January 23: Symmetry for a 2-port.

1978-5

January 24: An invariant wave equation on the $PSL_2(\mathbb{Z})$ tree. Characteristic impedance of a 2-port.

January 25,26: Reverse for 2-ports. On an electrical system on the $PSL_2(\mathbb{Z})$ tree, invariant under the group action.

January 27: Deriving Lee-Yang electrically.

January 28: Deriving the relation between impedances at the ports of an n -port.

1978-6

January 29: On the equation $z^3 - \frac{\alpha}{\gamma}z^2 + \frac{\alpha}{\gamma} - 1 = 0$ where $\alpha, \gamma \in \mathbb{R}$ and $\alpha^2 - \gamma^2 = 1$. The Hilbert space picture for circuits for a finite resistance network.

January 30,31: On the fundamental solution for $\Delta = \frac{1}{r^2} \frac{\partial}{\partial r} r \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2}$ in the plane, $\Delta u = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial u}{\partial r} \right) + \frac{1}{r^2 \sin \phi} \frac{\partial}{\partial \phi} \left(\sin \phi \frac{\partial u}{\partial \phi} \right) + \frac{1}{r^2 \sin^2 \phi} \frac{\partial^2 u}{\partial \theta^2}$ in three dimensions and $\Delta = \frac{d^2}{dx^2}$ on \mathbb{R} . Geometry of the UHP.

1978-7

February 1: On $\frac{\partial^2 \tilde{u}}{\partial t^2} = \Delta \tilde{u} = y^2 \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \tilde{u}$ in the UHP.

Geometry of the non-euclidean disc.

February 2,3: More on the geometry of Δ in the UHP.

February 4: The spectrum of Δ in the UHP. Eisenstein series. Note by Harald that $\zeta(2s)$ appears in the Eisenstein series instead of $\zeta(s)$.

1978-8

February 5: Attempt at automorphic Dirac equation. Maxwell's equations. Dirac style version of scattering in automorphic setting.

February 6: To construct an invariant self adjoint operator on the UHP starting with $\bar{\partial}$.

February 7: Eisenstein series of weight k . Continued fraction expansion and the $PSL_2(\mathbb{Z})$ tree.

February 8: On $\theta(\tau) = \sum_{n \in \mathbb{Z}} e^{i\pi n^2 \tau}$.

February 10: Evaluating $\sum_{0 \leq -\frac{d}{c} < 1} \frac{1}{c^s}$ where $-\frac{d}{c} \in \mathbb{Q}$ is in lowest terms.

February 11: Another version of the Γ' tree.

February: More on the $PSL_2(\mathbb{Z})$ tree.

February 13: Determining the spectrum of the Laplacian in the $PSL_2(\mathbb{Z})$ tree where $\delta f(x, y) = f(x) - f(y)$, $\delta^* g(x) = \sum_y g(yx)$ and $\Delta = \delta^* \delta$.

1978-9

February 14: Calculations exploring $\Delta u = \lambda u$ on the modular tree.

February 15,16: Cohomology for a tree and finite subtree.

February 17: Dirichlet norm on a Riemann surface.

February 18: Harmonic extension from its boundary to a disc. Ends of a modular tree.

1978-10

February 19: Spectrum of Δ on the modular tree,

February 20: Ford circles. Calculations for Δ in $L^2(\Gamma/G)$ where $\Gamma = PSL_2(\mathbb{Z}) \subset G = PSL_2(\mathbb{R})$.

February 23: Analysis on G/K where $G = SL_2(\mathbb{R})$ and $K = SO_2(\mathbb{R})$.

February 25: Atiyah's L^2 index theorem.

February 26: Analogues of invariant differential operators on the modular tree.

February 28: On de Branges theory. Nevanlinna matrix.

March 2: Review equations of motion for strings.

March 4: On a Dirac system with a δ -function potential.

1978-11

March 5: Continuing analysis of Dirac system with δ -function potential.

March 11: Review of electricity facts.

March 12,14: Analysis for a closed symmetric operator on a Hilbert space.

March 16,17: Closed symmetric operators with deficiency indices $(1, 1)$.

March 18: On non-necessarily densely defined closed symmetric operators.

1978-12

March 19,20: More on closed symmetric operators of deficiency indices $(1, 1)$.

March 23: On the equation $-u'' + qu = w^2u$ and de Branges functions related to solutions.

March 25: De Branges filtrations for $L^2(\mathbb{R}, d\mu)$, $\int \frac{d\mu}{x^2+1} < \infty$, equipped with a symmetric operator with deficiency indices $(1, 1)$.

March 26: On the general theory for a closed symmetric densely defined operator of type $(1, 1)$ with no self-adjoint component. Review of Sz-Nagy theorem about contractions.

March 27: More on contraction operators and scattering theory.

March 28: Note on a symmetric operator of type $(1, 1)$.

1978-13

March 29: Automorphisms of the category of symmetric type $(1, 1)$ operators without self-adjoint component and associated contraction operators.

March 30: Partial isometries and Waldhausen's treatment of π_1 for a manifold and a codimension 1 submanifold.

March 31: Contractions and scattering operators.

April 1,2: More on partial isometries with deficiency indices $(1, 1)$.

1978-14

April 11,13: More on contraction operators.

April 17: Szego's theorem.

April 18: Applying Szego's theorem to a contraction on $L^2(S^1l, d\nu)$.

April 20: More on contraction operators.

April 21: Invariant subspaces of a partial isometry.

April 23: Riesz-Herglotz representation of a rational function and the Blaschke product representation of a rational function.

April 28-May 4: Filtration associated to a partial isometry and associated contraction.

1978-15

May 5,6: Scattering matrix associated to 2-ports. Example from Schrödinger's equation $-u'' + qu = k^2u$ where q has compact support.

May 7: Note on functions f analytic for $|z| < 1 + \epsilon$, and g such that $|f| = |g|$ on $|z| = 1$ with the same zeroes in $|z| < 1$.

May 9: More on 2-ports.

May 10,11,12: Notes on a hermitian of signature $(+, -)$ on 2 dimensional vector space. Three equivalent descriptions of 2-ports.

1978-16

May 13: On $T \in GL_2(\mathbb{C}[z, z^{-1}])$, a polynomial function of z such that $|z| \leq 1 \Rightarrow T^*PT \geq P$ and $|z| \geq 1 \Rightarrow T^*PT \leq P$.

May 14: Transfer matrices $T(z)$ whose only singularity is at $z = 1$.

May 15: Factorizing matrices in $SL_2(k[\lambda])$.

May 18: Bound states associated to $L = \left(\frac{d}{dx} + p\right) \left(\frac{d}{dx} - p\right)$.

May 19,20: Note on $L = -\frac{d^2}{dx^2} + q$ where $q(x) \rightarrow 0$ fast as $x \rightarrow \pm\infty$.

May 21: Bound states for half line cases. Removing bound states.

May 22: On $L = -\frac{d^2}{dx^2} + q$, $L\phi_\lambda = \lambda\phi_\lambda$, $\phi_\lambda(0) = 0$, $\phi'_\lambda(0) = 1$.

1978-17

May 24,25: Hilbert space picture explaining 1-ports in terms of scattering or reflection coefficients.

May 28: On $-u'' = k^2u$, $0 \leq x < \infty$, $u'(0) = hu(0)$.

May 31, June 1: Incorporating bound states into the theory of relations between 1-ports, abstract scattering, inner functions, orthonormal polynomials over S^1 and Schur development.

June 2,3: Line bundle on \mathbb{P}^1 associated to an inner function. Piecewise-smooth functions and the Riemann-Hilbert problem.

June 4: More on inner functions and Toeplitz Weiner-Hopf operator.

June 4: Organizing a discrete version of the half-line inverse scattering problem.

1978-18

June 6,8: On the Deift-Trubowitz trace formula starting with Feynman-Kac formula, discrete case.

June 9,10: More on 1-port scattering function.

June 11,12: Construction of Schur system.

1978-19

June 13,14: More on Schur sequence and recurrence relation.

June 15: On the Schrödinger equation $-u'' + qu = k^2u$ on $0 \leq x < \infty$. Relationship between Dirac system and Schrödinger equation.

June 16,17,18: Example where $\phi(x, k^2) = \cos kx + \gamma \frac{\sin kx}{k}$ satisfies $-\phi'' + q\phi = k^2u$, $\phi(0) = 1$, $\phi'(0) = \gamma > 0$.

June 19: Scattering function and Cayley transformation.

June 20,21: Factorizing $S : S^1 \rightarrow S^1$, and associating a scattering function to a measure on S^1 .

June 24,25: Dirac systems and Schrödinger DE's on $0 \leq x < \infty$.

June 26: The continuous analogue of a port.

June 27-30: The splitting $L^2(dx) = L^2(-\infty, 0) \oplus L^2(0, b) \oplus L^2(b, \infty)$ and the semigroup $T(t)$, $T \geq 0$ where $T(t)$ takes $f \in L^2(0, b)$ shifts it distance t and projects onto $(0, b)$.

1978-20

July 1: On the relation between contraction semigroups and their infinitesimal generators.

July 2: On a closed densely defined operator B such that $(Bu, u) + (u, Bu) \leq 0$ and its Cayley transform.

July 3: On a maximal dissipative operator B and the semigroup e^{tB} , $t \geq 0$.

July 4: Review the scattering function S for a discrete 1-port.

July 5-8: Note on a partial isometry of type $(1, 1)$ and contractions extending it.

July 9: Multiplication by z on $L^2(S^1)$. Connecting a pair of 2-ports. Continuous analogue.

1978-21

July 10,11: Putting together a port (\mathcal{H}, A) and a transmission line.

July 12: Review discrete scattering.

July 15,16: Mathematical description of 1-ports.

1978-22

July 18: Continuous analogue of 1-ports and Schur process.

July 21-23: Orthogonal projections in $L^2(dk/2\pi)$.

July 24: Constructing a port $(\mathcal{H}, U(t), e_{\text{out}}, e_{\text{in}})$ for $-u'' + qu = k^2u$, $q \in C_0^\infty(\mathbb{R})$.

July 25: Dirac equations corresponding to a Schrödinger equation.

1978-23

July 26: Discrete Schrödinger equation.

July 27,28: Existence of scattering matrix when $\sum |h_n|^2 < \infty$.

July 30: Discrete Schrödinger equations - example of non-uniqueness for h_n .

1978-24

August 1: Spaces $\mathcal{H}, \mathcal{H}_x$ for Dirac equation.

August 2: De Branges spaces as ports.

August 5: Point valuation formulas for a port.

August 9,10,12: Singular strings (point mass, massless stretch). Continuous analogue of Schur representation.

August 21-24: Sequence of orthonormal polynomials associated to a probability measure on S^1 and associated Schur system.

1978-25

August 26-31: More on a probability measure on S^1 and associated Schur system.

September 2-9: Proving that if the response function is analytic for $|z| \leq 1$ and of modulus < 1 then the Schur parameters decay exponentially.

September 13: Uniqueness of Schur coefficients given the reflection coefficient.

1978-26

September 14,15: Existence of a system $(\mathcal{H}, U, p_n, q_n)$ given a Schur sequence $\{h_n\}$.

September 18,19: Proving that $R \in (\ell^1)^\wedge$ implies that $\sum |h_n|^2 < \infty$.

September 21,22: Proving that if $R \in (\ell^1)^\wedge$ and $|R| < 1$ then $\sum |h_n| < \infty$.

September 23: Inverse problem, finding the Schur sequence belonging to a given reflection coefficient R .

September 24: More on Schur sequences.

September 25: Calculating the matrix spectral measure for a Schur system in terms of the reflection coefficients R_+ , R_- .

1978-27

September 30-October 12: More on Schur systems. Scattering for a Dirac system. Transfer matrices.

October 22,24: Deift-Trubowirz trace formula. Marchenko equation.

1978-28

November 4: Schrödinger equation $-u'' + qu = k^2u$ and Green's function for $L = \frac{d^2}{dx^2} + k^2$. Newton's proof that $1 - Gq$ has Fredholm determinant $A(k)$ defined by scattering $e^{ikx} \leftrightarrow A(k)e^{-ikx} + B(x)e^{ikx}$.

November 9: Calculating $\det \left(\left(k^2 + \frac{d^2}{dx^2} \right)^{-1} \left(k^2 + \frac{d^2}{dx^2} - q \right) \right) = \det(1 - Gq)$.

November 10,11: Automorphic wave equation.

November 12: Laplace transform method for solving $\frac{d^2u}{dx^2} = -Lu$, $u(0) = u_0$, $u'(0) = u'_0$. The wave equation $\frac{d^2u}{dx^2} = (\Delta + \frac{1}{4})u$ on $y \geq a$ with $u(x+1, y) = u(x, y)$ and $\frac{\partial u}{\partial y}(x, y) = 0$ on $y = a$.

November 14: Duistermaat-Guillemin results relating the spectrum of Δ to the length of closed geodesics.

1978-29

November 17: Applying Marchenko-Faddeev to the automorphic scattering data

$$R(k) = \frac{\hat{\zeta}(1+2ik)}{\zeta(1-2ik)} \text{ in } \text{Im} k > 0.$$

November 17,19: Lippmann-Schwinger integral equations.

November 21: Levine-Schwinger on defraction by an aperture in an infinite plane.

November 22: Weiner-Hopf approach to the Sommerfeld problem.

November 23: Note on plane waves.

November 24: Schwinger problem.

1978-30

November 22,26: Maxwell's equations inside a wave guide.

November 28-December 2: Schwinger problem $\Delta\psi + k\psi = 0$ with $\frac{\partial\psi}{\partial n} = 0$ on the boundary.

December 5,7: Lippmann-Schwinger equation.

1978-31

December 10: Three dimensional scattering for a spherically symmetric potential.

December 14-18: Three dimensional scattering for $(\Delta - V + k^2)\psi = 0$ where V has compact support.

1978-32

December 19,20: Understanding the significance of $1 - G_k^+ V : \text{Ker}(\Delta + k^2 - V) \rightarrow \text{Ker}\Delta + k^2$ for the scattering problem.

December 21: Dirac system $\frac{d}{dx} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} ik & h \\ \bar{h} & -ik \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$ on $[0, 1]$ where $u_1(0) = u_2(0)$.

December 22: More on $1 - G_k^+ V$.

December 23: Showing that Dirac and Schrödinger determinants have the same zeroes.

December 24: Are $\det \left(1 - \left(\frac{d}{dx} + ik \right)^{-1} h \left(\frac{d}{dx} - ik \right)^{-1} h \right)$ and $\det \left(1 - (\Delta + k^2)^{-1} (h' + h^2) \right)$ equal?

December 25: Obstacles in a wave guide.

December 26-28: Scattering by a sphere.

1978-33

December 29: Return to Schwinger problem $(\Delta + k^2)\psi = 0$, $\frac{\partial \psi}{\partial n} = 0$ on boundary.

December 30: Green's function formulas.

December 31: Scattering by a circle in the plane.

Contents 1979

1979-1

January 1,2: Obstacle scattering with Dirichlet boundary condition.

1979-2

January 4,5,6: Discrete scattering.

January 7: Continuous analogue interpretation of $\det(I - G_\zeta^- V)$ as a characteristic polynomial.

January 9: Analysis of $H = \frac{1}{i} \left(\frac{d}{dx} - V \right) = H_0 + iV$ as an operator on $L^2(\mathbb{R})$. Dirac scattering on \mathbb{R} .

January 10: Interpretation of $\det(I - G_\lambda^+)$ as a characteristic polynomial for the Schrödinger differential equation.

January 11: Schrödinger differential equation on \mathbb{R} and associated wave equation.

1979-3

January 13,14: Reconciling Lipmann- Schwinger and Lax-Phillips for the radial Dirac system.

January 15,16: On a Dirac system on $0 \leq x \leq \infty$, $\frac{d}{dx} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} ik & p \\ \bar{p} & -ik \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$, $u_1(0) = u_2(0)$, $p(x) = c\delta(x - \epsilon)$.

1979-4

January 23: Commutative algebra interpretation of $\det(I - G_z^+ V)$.

January 24-28: Defining $\det J$ when J is an endomorphism of a free rank 1 module over $\mathbb{C}[z, z^{-1}]$.

1979-5

January 25-31: Understanding \mathcal{D}/\mathcal{D}' for the continuous case when the wave equation is $\frac{\partial^2 u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - Vu$.

1979-6

February 2: On understanding Schwinger V, Phys Rev 93, 615.

February 3: More on the Lipmann-Schwinger determinant.

February 4-8: Green's functions for the Klein-Gordon equation.

February 9: State vector for the Schrödinger picture. Schwinger's formula relating a vacuum expectation to a determinant.

February 10: Propagation approach to the ordinary Schrödinger equation.

February 11: Review of S -matrix formulation. Understanding the interaction Hamiltonian in the case of a Dirac field perturbed by an external EM field.

1979-7

February 13,15: Comments on analogies between interacting quantum fields and second quantization applied to the Klein-Gordon equation. Quantizing a classical field theory derived from a Lagrangian.

February 17: Segal's viewpoint on quantum field theory.

February 18: Viewpoint applied to the Klein-Gordon equation.

February 19,20,21: Understanding the S -matrix on the classical level. Classical mechanical system picture. Heisenberg picture.

February 22: Understanding the field equation $\phi'' = -(-\Delta + m^2)\phi$ as a continuous system of coupled harmonic oscillators.

1979-8

February 23,24: On a coupled harmonic oscillator with Hamiltonian $H = \frac{1}{2} \left(\sum_i p_i^2 + \sum_{i,j} q_i V_{ij} q_j \right)$.

February 27: On understanding Schwinger's transformation functions.

March 2: On the Hamiltonian $H = a^* a - \frac{1}{2} \epsilon(t) q^2$.

March 3,4: Schwinger's variational principle.

March 6,9: On $H = \frac{1}{2} [p^2 + (\gamma q)^2 - q \epsilon q]$.

March 10,11: Quantizing the Dirac differential equation: $\left[\frac{1}{i} \frac{\partial}{\partial t} + \alpha \frac{1}{i} \frac{\partial}{\partial x} + m\beta \right] \psi = 0$.

March 12: Observation on a complex vector space equipped with a hermitian inner product.

1979-9

March 13-16: Fermion quantum mechanics and Clifford algebras.

March 18: Classical action and quantum mechanical propagation. Hamilton-Jacobi partial differential equation.

1979-10

March 20: Morse theory from the viewpoint of classical mechanics. Gauge theory and a review of Bott's lectures.

March 22-26: Quantizing the Dirac equation.

March 27-30: Schwinger's sources. Understanding $(\lambda - H_0)^{-1}(\lambda - H) = 1 - G_0(\lambda)V$ where H_0 is the Dirac operator and $H = H_0 + V$.

April 7: Lax-Phillips semi-groups.

1997-11

April 12: Classical magnetism model, Ising model, Heisenberg ferro-magnetism model, quantum mechanical model.

April 13: Gauge theory.

April 14: On Feynman path integrals.

April 15,16: Path integrals from Abers-Lee.

April 17: A question about discrete analogues of quantum mechanical motion on the line.

April 19: Solvable models in algebraic statistical mechanics.

April 24: Review of statistical mechanics.

April 22-24: Understanding some thermodynamics. Maxwell-Boltzman distribution.

1979-12

April 25: Analysis for a Hamiltonian which depends only on $k = \sum_{i=1}^N s_i$, giving partition function $\sum_{k=-n}^n \binom{2n}{n+k} e^{-\beta H(k)}$ where $2n = N$.

April 26: Simons-Griffiths limiting process.

April 27: Entropy and free energy.

April 29: Magnet model and spin waves.

May 3,4: BCS model.
 May 5: Bose gas. RH for finite fields.
 May 6: Ising model again.
 May 7: Classical grand canonical ensemble.
 May 8: Review box and reservoirs.
 May 10: Idea for a model of a liquid using K -theory.
 May 13,15,16: Maxwell's equations using differential forms.
 May 17: On relativity.
 May 19: On special relativity.
 May 20: Analysis of a geodesic flow twisted by a 2-form.
 May 25,26,27: Magnetism. Review of electrodynamics. Magnetostatics. Yang-Mills.

1979-14

May 28,30,31: Lagrangian approach.
 June 1-5: Solving first order PDEs and Hamiltonian-Jacobi PDEs.
 June 8: On the calculus of variations and Feynmann integrals.
 June 9: Geometric optics from the wave equation viewpoint.
 June 10: Solution to $\frac{\partial S}{\partial t} + H(t, q, \frac{\partial S}{\partial q}) = 0$. Solution to Hamilton-Jacobi equation $L(q, \dot{q}) = |\dot{q}|$.
 June 11: Phase and group velocity. Return to solving $\frac{\partial S}{\partial t} + H(t, q, \frac{\partial S}{\partial q}) = 0$ where H does not depend on time.
 June 12: Huygen's wave-front approach to classical mechanics.
 June 13: Relativistic motion of a particle.
 June 15,16: Wave front description for classical relativistic mechanics.

1979-15

June 17: Maupertuis principle.
 June 18,19,21: Getting a second order differential equation $\ddot{q} = F(t, q, \dot{q})$ into Hamiltonian form.
 June 22: On wave fronts and energy momentum. Relativistic motion.
 June 23: More on relativistic motion of a single particle.
 June 24: Summary of particle motion. Quantization.
 June 25: On understanding the motion of a charged particle in an external electro-magnetic field.

1979-16

June 26,27,28: On understanding scattering by an external electro-magnetic field of compact support in space time.
 June 30: On the $\bar{\partial}$ -problem associated to $L^2(\mathbb{R}^n)$ where q_i = multiplication by x_i , $p_i = \frac{1}{i} \frac{\partial}{\partial x_i}$. Review harmonic oscillator with perturbation $H = \frac{1}{2}p^2 + \frac{1}{2}q(w^2 + \epsilon)q$. Phonons in a crystal.
 July 4: Scattering of a slow neutron by a crystal. Review of potential scattering, the S -matrix, Lipmann-Schwinger.

July 5: Scattering matrix when the potential V is spherically symmetric.

1979-17

July 6: Conventions concerning the Ricatti-Bessel functions. More on scattering theory for a spherically symmetric potential.

July 7,9: On the T -matrix.

July 13: More on phonons.

July 14,15: Perturbing the potential energy term of the oscillator. Feynmann diagrams and integrals.

July 17,18: How diagrams label terms in the perturbation expansion for the ground state energy.

July 19: Review the perturbation problem $H = H_0 + V$ of the Hamiltonian.

July 22: On $H = H_0 + V$ where $H_0 = \sum E_k a_k^* a_k$ and $V = \sum V_{kl} a_k^* a_l$.

1979-18

July 22: More on Schwinger - finding the S -matrix.

July 26,27,28: A forced harmonic oscillator.

July 29: Schwinger and sources in the fermionic situation.

July 30,31: Review path integrals.

August 1: Understanding Green's function for an oscillator. Linear response (Kubo).

August 2: Perturbative expansion of the Green's function.

August 3: On the 1-particle Green's function.

1979-19

August 10: Perturbing an oscillator by a source, translation action and holomorphic representation. Note on uncertainty and coherent states. Quantizing the Klein-Gordon field $\ddot{\phi} = -(-\Delta + M^2)\phi$.

1979-19

August 11,12: Formulas for diagonalization of the Hamiltonian for scalar phonon oscillator on a finite abelian group and quantizing the Klein-Gordon field.

August 13: Metaplectic representation. On $H = \frac{1}{2}p^2 + \frac{1}{2}w^2q^2 + \frac{1}{2}\epsilon(t)q^2$.

August 15: To compute $e^{\frac{aD^2}{2}} e^{\frac{bx^2}{2}}$ and apply it to the oscillator $H = \frac{1}{2}p^2 + \frac{1}{2}(wq)^2 + \frac{\epsilon}{2}q^2 - Jq$.

1979-20

August 17: Fermion algebra. Slater determinants. 2nd quantization. Degenerate electron gas.

August 18: Degenerate electron gas.

August 19: Many particle system of fermions. Leyman representation.

August 20: Many body problem.

August 23: Notes and comments on the simple harmonic oscillator subjected to an external perturbation.

August 24,25: Expansion of $e^{t(A+B)}$. Adiabatic formalism (as in Schweber's book). Gelmann-Low formula for general energy shift.

August 26: Review adiabatic formalism.

August 27: Evolution of eigenstates.

1979-21

August 28,31: Calculations for $A_0 = -\frac{d^2}{dx^2} + m^2$ and $A = -\frac{d^2}{dx^2} + M^2 + \epsilon(x)$, ϵ small. Review quantizing Klein-Gordon field.

Septemebr 1: Does the adiabatic approach work for a finite dimensional operator?

September 3: To understad the effect of a single impurity atom on an electron gas.

September 6: Scattering for a spherical potential revisited.

September 9,12: Computing the transformation function $\langle q'|U_J(T,0)|q \rangle$ for $H = \frac{1}{2}p^2 + \frac{1}{2}(wq)^2 - Jq$ where J is supported in $[0, T]$.

September 15: Expressing the S -matrix in terms of time-ordered vacuum operator values.

September 16: Green's function for a system with a time dependent Hamiltonian.

1979-20

September 17-23: On "in" and "out" operators. Review thinking of quantum mechanics as a bundle on Hilbert spaces over a time axis. Calculating the S -matrix.

September 24: Diagrams in energy-momentum notation.

September 28: Melrose's problem on the propagation of singularities for $\partial_t^2 u = (\partial_x^2 + x^2 \partial_y^2)u$.

September 29: To construct the Green's function for $-\partial_x^2 + \eta^2 x^2$.

1979-23

September 29-October 9: Victor's problem: When K be a strictly convex compact body in \mathbb{R}^n , containing 0 in its interior, $N(t)$ = number of lattice points in $tK = \sum_{x \in \mathbb{Z}} \xi_K(\frac{1}{t}x)$, understand $N(t)$ as $t \rightarrow \infty$. Van der Corput situation. Singularities of $K(x, t) = (2\pi)^{-n} \int d\xi e^{i(x\xi - tQ(\xi))}$

October 10: On $-\Delta + m^2 + V$ where V is small.

1979-24

October 12,13: Instantons - Colman's lectures, and related calculations. Review of some solitons.

October 14: Double well potential.

October 16: Electrodynamics review.

October 17: Dirac equation. Relativistic notation. Field equations for electrodynamics.

October 18: Variational problems occuring in Hamilton's principle.

1979-25

October 19: Contribution of multiple instantons to the amplitude $\langle x, |e^{-T\frac{H}{\hbar}}|x' \rangle$ as $T \rightarrow \infty$.

October 20: WKB answers by instanton methods.

October 21: DHN papers (Dashen, Hassacher, Neveu) Phys Rev D11 (1975), 3424.

October 26: Path integrals $\langle x|e^{iTH}|x' \rangle = \int \mathcal{D}x(t)e^{-S}$.

October 27: Volterra transform methods of Gelfand and Yaglom. Feynmann integrals.

October 28: Two ways of constructing a classical semi-classical approximation to the amplitude $\langle x|U(T,0)|x' \rangle = \int_{x(0)=x', x(T)=x} \mathcal{D}x(t)e^{i\frac{1}{\hbar} \int_0^T (\frac{1}{2}\dot{x}^2 - V)dt}$.

October 29,31: On solving $(-\frac{1}{2}\Delta + V)\psi = E\psi$ by the WKB method.

November 2: Understanding the approximation to a quantum problem obtained by expanding around a classical solution.

November 3: Legendre transformation. Maxwell-Boltzman distribution.

1979-26

November 4-10: Understanding the quantum mechanics of $H = \frac{p^2}{2} + V(q)$ where $V = \frac{1}{2}w^2x^2 + gx^4$.

November 14: Victor's lectures on Tauberian theorem.

November 16: Wiener Tauberian theorem. Review of the ζ -function, Prime Number theorem. Exponential integral function.

November 18: How Tauberian results are used in spectral theory.

1979-27

November 21: Mumford's theorem on resolving singularities of an isolated singular point on a surface.

November 25,26: Mumford's theorem concerning real symmetric matrices $A = (a_{ij})$ with $a_{ij} \leq 0$ for $i \neq j$. Relation between Mumford's theorem and Frobenius theory for matrices with positive entries. More Tauberian results from Victor's course.

November 28: Amit's account of the Ising partition function.

November 30: Ising model partition function.

December 1,3: Computing the partition function $\text{tr}(e^{-\beta H})$ for the 1-dimensional harmonic oscillator $H = \frac{p^2}{2} + \frac{1}{2}w^2q^2$.

December 5: On understanding free field theory.

December 6: Victor's lecture on Szego's theorem (Widom's proof).

1979-28

December 7,8: Analogy between Ising model partition function and field theory generating functions.

December 9-15: Computations for the ϕ^4 -theory using Feynmann diagrams. Weiss theory for Ising model.

December 16: Free graphs and Legendre transformations.

1979-29

December 24: Correction terms for Stirling's formula by Feynmann graphs.

December 26: On the vertex function.

December 27: Fermi gas.

December 28: Fermi's version of thermodynamics formulas.

December 29,30: To find a good model for interacting fermions.

December 31: Heisenberg chain.

1979-30

Late December conversations with Kostant.

Contents 1980

1980-1

- January 1: Spin waves (continued).
- January 4: Schwinger's auto-commuting c -numbers. Boson case. Green's function using Dyson formula. Path integral description for $\text{tr}(U(\beta, 0))$
- January 5,6: Computing $\text{tr}(U(\beta, 0))$ for $H = wa^*a + Ja + \tilde{J}a^*$. Fermion case. Fourier transform.

1980-2

- January 11,12: About the KdV equation.
- January 13: Marchenko equation.
- January 14: Computing trace invariants for $D^2 + q$ which are invariant under the KdV flow.
- January 15,16: Computing the diagonal part of the kernel for e^{-TH} , where $H = \frac{1}{2}p^2 + V$ and T small.
- January 17,18,19: On understanding $e^{t(D^2+q)}$ for t small. Mellin transform.

1980-3

- January 20: On the resolvent $(K^2 - D^2 + V)^{-1}$.
- January 22: Deriving the formula $\langle A \rangle = \text{tr}(Ae^{-\beta H})/\text{tr}(e^{-\beta H})$ using a microcanonical ensemble. Relating resolvent and heat kernel.
- January 25: Fermion integration. Evaluating $\frac{\int e^{\frac{1}{2} \sum a_{ij} v_i v_j} \prod v_i}{\int e^{\frac{1}{2} \sum a_{ij} v_i v_j}}$, where (a_{ij}) is skew-symmetric and non-singular. Fermion path integrals and Feynmann diagrams for interacting fermions.
- January 26: Constructing a path integral involving fermionic integration associated to the Hamiltonian $H_0 = \sum u_k a_k^* a_k$.
- January 27,28: On understanding Feynmann diagrams and functional integrals for fermions.

1980-4

- January 29,30,31: Calculating $\frac{\text{tre}^{-\beta H}}{\text{tre}^{-\beta H_0}}$. Ground energy shift.
- February 1: Adiabatic switching.
- February 2: Goldstone diagram.

1980-5

- February 3,9: Poisson distribution. Oscillators.
- February 10: Review electro-magnetism. Fermions and continuous spectrum.
- February 11: Dirac equation in 2-dimensional space time with an external stationary electro-magnetic field.
- February 13-18: Second quantization.

1980-6

- February 19: More on second quantization.
- February 20-24: On a gas of independent fermions.
- February 27: Formulas for $\Lambda^*(\mathbb{C}^p \oplus \mathbb{C}^{n-p})$.

March 2: Fredholm formulas for $(1 - \lambda k)^{-1}$.

March 3,5: Weinberg's analysis of an N -body system.

1980-7

March 7,8,9: Weinberg quasi-particle, Schmidt method.

March 10,12: Response of an electron gas to an external electromagnetic field.

March 14,15: Spin and the Dirac equation.

March 16: Kubo formula.

March 17: Response of an independent particle to a weakly varying field.

1980-8

March 19: On the Lehmann representation.

March 21: Review scattering for potential $V(r)$.

March 23: Scattering on the line for a potential $V(|x|)$. More on Weinberg and an independent fermion gas.

March 24,26: X-ray singularity, single impurity problem.

March 28: Distribution $(x_+)^{s-1}$ on \mathbb{R} . Renormalization group Kadanoff idea.

March 29, 30: Return to studying independent fermion gas perturbed by an external potential.

1980-9

April 11: Dielectrics.

April 12: Calculating the linear response of an electron gas to an external charge distribution.

April 14: Speed of sound. Pressure of fermion gas.

April 16: Fermi gas in a box.

April 18,20: Sound waves in a Fermi gas at 0 temperature. Plasma frequency from Feynmann lectures.

April 21: On the effective potential method for describing the interacting electron gas.

1980-10

April 22: Understanding density fluctuation in an electron gas.

April 26: De Bohr theory of the hydrogen atom.

April 27: Derivation of the TH approximation to static linear response.

April 30: Calculation of $\mathcal{P}^2(q, w)$ describing fluctuation in a non-interacting fermi gas.

May 3: Calculation of the speed of sound waves in a fermi gas for dimension d .

May 7: Specific heat of a fermi gas. Sackur-Tetrode formula.

May 9: Interacting fermi system.

1980-11

May 10: Interacting fermi system. Debye specific heat. Planck's radiation law.

May 17: Spin waves as an interacting Bose system. Magnetic monopoles.

May 20: Angular momentum.

May 21: Pauli matrices.

May 22,23: Spin up and spin down. Particles in a box.

May 24: Review of Ising model with N spins.

1980-12

May 30: Thermodynamic relations. Clausius-Clapeyron equation.

May 31: Entropy of mixing two ideal gases at the same temperature.

June 1: Ising model with energy $E(s) = -\mathcal{H} \sum s_i - \frac{1}{2} \sum K_{ij} s_i s_j$.

June 3,4: Derivation of Boltzmann law and heat reservoir.

June 8,13,15; Review first order pde's and heat flow on (t, p, q) -space described by Hamilton's equations.

1980-13

June 17: Heat reservoir. Thermo-potential. Clausius inequality.

June 18: Example of irreversible change: Joule experiment.

June 20,21: Sackur-Tetrode formula from the vapour pressure of a crystalline solid.

June 22,23: Vapour pressure of a crystal.

June 24,25: Chemical potential approach.

June 26: On solid \leftrightarrow vapour when the solid is described by Einstein or Debye model.

June 27: Equilibrium for ideal gas reaction.

1980-14

June 29: Linearized versions of Hamilton's equations.

July 2: List of units. Wheeler's constant.

July 3: Understanding first and second variations associated to least action.

July 4: An action functional associated to $H = \frac{1}{2}p^2 + \frac{1}{2}w^2q^2$.

July 5,6,7: Schrödinger equation for a scalar wave function.

July 9: Canonical transformations.

July 10: Free motion and WKB formula. Description of next project.

July 12: Deriving FD and BE distributions using dominant term instead of grand partition equation.

1980-15

July 13: Ferromagnetism model. Weiss equation.

July 16: RG = renormalization group.

July 17: Mark Reynold's problem - Gaussian integrals.

July 18,20: Weiss model.

July 21: Rubber elasticity.

July 22: Einstein theory of Brownian motion.

1980-16

July 23: Virial expansion for imperfect gas. Review of BE gas.

July 24,25,26: Imperfect gas virial expansion.

July 27: Vertex functions and 1PI graphs.

July 28: Monotonic gas.

July 29: Mayer cluster expansion.

1980-17

July 30,31: Green's function for the classical grand ensemble.

August 1: Functional differentiation. Grand partition function.

August 2: Classical grand partition function with variable activity.

August 3: Infinite symmetric product. Grand partition space $SP(X)$.

August 4: Green's functions for grand partition function.

1980-18

August 5,6: More on Mayer cluster expansion. Ornstein-Zernike equation.

August 7: Van der Waals forces. Derivation of virial expansion.

August 9: Pseudo-potential.

August 10: Brownian motion on the line with potential $V(x)$.

August 11: Linear response theory.

1980-19

August 13: Monoid of probability measures on \mathbb{R} . Infinitely decomposable probability measures on \mathbb{R} . Poisson processes. Dielectric behaviour.

August 14: Neutron scattering by a crystal.

August 16: Motion of Gaussian wave packets in time.

August 17: Remarks on scattering by a potential from a classical point of view.

August 18: Feynmann's inequality.

August 19,20: Effective potential for quantum motion on the line.

1980-20

August 21: Wick ordering.

August 22: Review of S -matrix for the harmonic oscillator. Review of path integral formulas.

August 23,24: Master equation. Time dependent perturbation theory.

August 25,26: Langevin approach to Brownian motion.

August 27: Fokker-Planck equation. Wiener-Khinchin theorem

1980-21

August 29,30: Inelastic scattering situation. Simple example.

September 1: Scattering by a system that can go into an excited state. More on the simple example.

September 2: Nyquist relation.

September 3: A transmission line as a simplified model of an EM field.

1980-22

September 4: More calculations with a transmission line. Nyquist formula.

September 5: Weiner-Khinchin theorem. Notes on two ideas.

September 6: Note on a 1-port of inductances and capacitances.

September 7: Transmission line notes. Feynmann's version of a noise signal from a Geiger counter.

September 7: More transmission line calculations.

September 12: Two dimensional QED.

1980-23

September 13: Plain wave solution to Maxwell's equations.

September 14,15: Quantizing the EM field as a bunch of independent harmonic oscillators. Absorption and emission of photons by a charges particle.

September 17,19,20: Transmission line analogue of the EM field.

September 21,22: Model of absorption and emission of photons.

September 24: String model.

September 26,27: More on the model for emission and absorption of photons.

1980-24

October 1,3: Review forced harmonic oscillator.

October 5: Einstein's A,B. Planck's context.

October 6: More on the enforced oscillator.

October 10,11: Time dependent approach to scattering to first order.

October 12,13: Calculating the S -matrix using frequency.

October 20,22: More on the enforced oscillator.

October 24,25: Scattering theory calculations.

1980-25

October 26,27,November 3: More on the forced oscillator.

November 7,8,9: On the S -matrix for a quadratic pertubation of a harmonic oscilator.

November 10: When is the operator U defined by $\langle e_\mu|U|e_\lambda \rangle = e^{\alpha \frac{\mu^2}{2} + \bar{\mu}\beta\lambda + \gamma \frac{\lambda^2}{2}}$ unitary.

November 12: Finite time perturbation of a simple harmonic oscillator.

November 14,15: On the S -matrix for a perturbed oscillator.

November 16: On the operator U given by $\langle q|U|q' \rangle = \det(2\pi\beta)^{-\frac{1}{2}} e^{i(\frac{1}{2}q \cdot a q - q' \cdot \frac{1}{\beta} q + \frac{1}{2}q' \cdot c q')}$. Formulas in the holomorphic representation.

1980-26

November 19,21: Calculations related to the holomorphic representation.

November 22,23: On the perturbed harmonic oscillator $H = \frac{1}{2}p^2 + \frac{1}{2}(w^2 + \epsilon)q^2$ where $\epsilon(t)$ decays as $|t| \rightarrow \infty$. Green's function and the S -matrix.

November 26,29,December 1: On a perturbed wave equation $\ddot{\psi} + (-\Delta + V)\psi = 0$ on $[x,\infty)$.

December 5,7: More on the problem of emission and absorption.

December 10: More on the S -operator fro a simple harmonic oscillator.

December 11: On the Green's function satisfying $[\partial_t^2 + (-\partial_x^2 + V)]G_t(x, x') = \delta(t)\delta(x - x')$.

1980-27

December 12: On the simple harmonic oscillator with finite time perturbation applied to the infinite dimensional oscillator represented by the wave equation and the perturbation by the wave equation with potential.

December 14,15: On the fermion analogue of the perturbed oscillator.

December 19: On two oscillators described by $H_0 = w_1 a_1^* a_1 + w_2 A_2^* a_2$ and coupling between them $H_{\text{int}} = \varepsilon q_1 q_2$, $q_i = \frac{a_i + a_i^*}{\sqrt{2w_i}}$.

December 21: Review Nyquist's relation.

December 22: On a line of density λ , tension λ connected to an oscillator.

December 23: More on finding a simple model of emission and absorption.

December 26: More on a line of density λ , tension λ connected to an oscillator.

December 27: Examples of light strings.

Contents 1981

1981-1

- January 1: Harmonic oscillator $H = w_0 b^* b + \sum_k (w_k a_k^* a_k + b^* \bar{\gamma}_k a_k + a_k^* \gamma_k b)$ and the Lee method.
- January 3: Lee model.
- January 8: Gauge theory over a Riemann surface.
- January 9: Atiyah-Bott theory of Yang-Mills over Riemann surface.
- January 10,11: Yang-Mills theory.
- January 12: On S^4 as a projective line bundle over the quaternions.

1981-2

- January 13: Gauge theory with a matter field on \mathbb{R}^2 - abelian Higgs theory.
- January 14-28: Inverse scattering theory, starting with a review of 1978 work.

1981-3

- January 29-February 6: Review of J -matrices and KdV flows.
- February 7,8,9: Hartree-Fock approximations in an interacting Fermi gas.
- February 15,16: Mean field theory.
- February 17: Derivation of van der Waals equation by Reif's argument.
- February 26: Many body formalism.

1981-4

- February 27: Many body problem.
- February 25,27: Review the classical gas: $\mathcal{Z} = \sum_n \frac{1}{n!} \int \Pi_1^n z(q_i) dq_i e^{-\beta U_n(q_1, \dots, q_n)}$.
- March 2: Quantum gas.
- March 4: Notes on measures and generating functions.
- March 5: Review Wiener-Khinchin.
- March 7: Gaussian processes and Markov processes.
- March 9: Poisson processes on the line.
- March 11: Levy-Khinchin.
- March 14: Return to mean field theory. Weiss theory.
- March 18,20,22: Applying dominant term idea to ideal gas.

1981-5

- July 5: Review the structure of rings like $\oplus_n H_*(BG_n)$ for the symmetric groups and finite general linear groups.
- July 7: On the ring homomorphism $K(X) \rightarrow \text{Hom}_{\text{rings}}(\oplus R(\Sigma_n)^\vee, K(X))$. Green's paper on representations of $G_n = GL_n(\mathbb{F}_q)$. On $R(G)$.
- July 8: On $R(G)$ where G is a finite group.
- July 9: Representations of G_2 .

July 10: On representations of $G_a \times G_b$.

July 11: Review of Hecke algebras.

July 13: Why Green's multiplication is commutative.

July 15,16: On discrete series representations.

July 19: On representations of $G_n = GL_n(\mathbb{C})$. More on commutativity of the Green's product.

July 27: Review of Deligne's proof of the $\lambda^i \lambda^j$ identities.

1981-6

August 16: Review of buildings and Lie groups.

August 19: On Atiyah's Arbeitstagung talk on convexity and commuting Hamiltonians, including a proof of a theorem of Kostant.

August 20,21: Proof of Kostant's theorem for several special cases.

August 23,24: Counter-example showing $T \backslash U_n / T \neq$ doubly stochastic matrices.

August 25: Review $\Omega(K)$ and Laurent polynomials where K is a compact connected Lie group.

August 28: On the natural symplectic structure of $\Omega(K)$.

August 29: Kirillov setup.

August 30-September 4: Analysis of $\mathcal{K} = \text{maps}(\phi : S^1 \rightarrow K)$ and $\chi = \text{space of connections on the bundle} = \text{maps}(S^1 \rightarrow \text{Lie}(\mathcal{K}))$.

September 5: Analysis of a sign problem for the action of a group on a manifold.

September 6: Gauge theory on S^1 .

September 6,9: Representation theory of the gauge group.

September 10: Description of the irreducible representations of $S^1 \ltimes (\mathbb{Z} \times S^1)$.

September 12,13: Representations of the gauge group associated to $SU(2)$.

1981-7

September 14: More on the representations of the gauge group associated to $SU(2)$. Jacobi's identity: $\prod_{n \geq 1} (1 + q^n x^{-1}) \prod_{n \geq 0} (1 + q^n x) \prod_{i=1}^{\infty} (1 - q^i) = \sum_{n \in \mathbb{Z}} q^{\frac{n(n-1)}{2}} x^n$.

September 17: Kac-Moody Lie algebras.

September 18: Classification of homogeneous symplectic manifolds for a simply connected Lie group.

September 20,21,22: Understanding the canonical extension of $SL_n(\mathbb{C}[z, z^{-1}])$ given by the tame symbol. Holomorphic function representation of the canonical commutation relations. Shake's theorem. Cocycle of the central extension of the symplectic group given by the unitary operations preserving V .

September 24,25: On the metaplectic covering of $SL_2(\mathbb{R})$ using the real representation of $p = \frac{1}{i} \frac{d}{dx}$ and $q = x$ on $L^2(\mathbb{R})$.

September 27,28: Simple calculations and questions for $SL(F)$ where F is a field.

October 1: On $SL_2(F)$ where F is the Laurent series field $F = \mathbb{C}[[z]][z^{-1}]$. Loop algebras.

October 2: On the Kac-Moody Lie algebra which covers $sl_2[z, z^{-1}]$.

October 4: Understanding the Kac-Weyl character formula for the loop algebras.

1981-8

October 5: On finding a line bundle on \tilde{K}/\tilde{T} where $K =$ algebra of maps $S^2 \rightarrow K$, and \tilde{K} is the central extension.

October 6: Irreducible representations of $SL_2(\mathbb{R})$. Holomorphic representation of $\widetilde{SL_2(\mathbb{R})}$. Ideas about representations of loop groups. Is there an analogue of the metaplectic representation of local fields like \mathbb{Q}_p ?

October 7: Remarks on the group $\text{Map}(S^1, S^1)$, Canonical central extension of $SL_2(\mathbb{C}[z, z^{-1}])$ restricted to the subgroup isomorphic to $\mathbb{Z} \times \mathbb{C}^*$.

October 8: On a central extension of $SL_2(F)$ where F is the local field of Laurent series $F_q([T])$ over a finite field F_q .

October 9: On the characteristic Kac modules. Remarks on representations of $F = \mathbb{C}[z, z^{-1}]$ and F^* .

October 10: On central extensions of the Lie algebra F and the group F^* .

October 11: Poisson summation formula and compatibility of Heisenberg extensions of $\mathbb{Z} \times S^1$ and $\mathbb{R} \times \mathbb{R}$. Dilogarithms.

October 12: Atiyah's L^2 index theorem.

October 14: Remarks on the alternating pairing on maps $S^1 \rightarrow \mathbb{C}^*$ given by the dilogarithm pairing.

October 15: Vertex operators.

October 18: Kac-Frenkel formula for the vertex operator $X(\gamma, z)$. Dual resonance modules.

October 21: More on vertex operators.

October 22: Vertex operators for coherent states.

October 29: A unified way to think about Kan's $G\overline{W}$ functions, and bar and cobar operations in rational homotopy theory. Return to vertex representation. Physics of gauge fields.

October 26: Gauge fields.

October 27: What is a joint representation of \mathcal{G} , the gauge group, and χ the space of connections and a torsor for $\text{Lie}(\mathcal{G})$.

October 28: Review proofs of periodicity and indices.

October 29: Quantization of the EM fields.

October 30: On understanding the Feynmann approach to the quantization of the EM theory.

1981-9

November 1: General remarks on quantizing objects in topology.

November 2: General Feynmann approach illustrated in one degree of freedom.

November 5: Connes pseudo-torus and a generalization.

November 6: Comment on constructing a type II algebra.

November 8: Connections between ζ functions, cohomology and K -theory.

November 9: Comments on finding a uniform viewpoint for K -theory with Adams operations and Langlands theory with Hecke operators.

November 17,18: Return to studying instantons.

November 22,23: More on vertex operators.

November 24,26: On constructing irreducible representations of a loop group as holomorphic functions of a line bundle over an orbit.

November 27: On constructing sections of the canonical line bundle over the space of lattices.

November 28,29: Algebraic version of fermion Fock space.

November 30: Comment on constructing the central tame extension of GL_n for local field and number theoretic cases.

December 1,2: Fock space and scattering theory.

1981-10

December 4,5,6: Abstract algebra associated to Fock space calculation.

December 7: Japanese solution to the Riemann-Hilbert problem.

December 9,10: Representations of central extensions of a Lie algebra.

December 12: Review KdV equation.

December 14,15: Computing $\langle 0|\tilde{S}|0\rangle$ for $S : S^1 \rightarrow S^1$,

December 16,17: Factorization of the S -matrix. Marchenko equation method.

December 20: Marchenko equation with bound states.

December 25,26: Review analysis of $\frac{d}{dx} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} ik & p \\ \tilde{p} & -ik \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$ where p, \tilde{p} decay fast as $|x| \rightarrow \infty$.

December 29: On $[-\partial_x^2 + q]u = k^2u$ in the case where there is no reflection.

December 31: On soliton solutions of the KdV equation.

Contents 1982

1982-1

- January 3,4: Riemann-Hilbert problem.
January 4, 6: Soliton solutions of KdV. Baker-Akhiezer function.
January 7: Scattering theory in $L^2(S^1)^d$.
January 10, 11: Notes on Krichever theory (Mumford paper).
January 12: Formal expressions for the resolvent of $L = \partial^2 + q$.

1982-2

- January 14: Riemann-Hilbert problem - Japanese viewpoint.
January 15: Calculations in $V = \mathbb{C}[z, z^1]$ with $V_- = z^{-1}\mathbb{C}[z^{-1}]$, $V_+ = \mathbb{C}[z]$; $\Lambda(V; V^+) = \Lambda(V_-) \otimes \Lambda(V_+)^{\vee}$. Vertex operators.
January 17: $V = L^2(S^1)$. Fock space calculation.
January 19: Continuing the calculations of January 15. Discussion of derivations.
January 15: Note on Dirac system.
January 24: On the modified $\Lambda(V)$ where V is a finite dimensional vector space over a function field of a curve. Duality.
January 27, 28: Fredholm theory.
January 29: Factoring the S -matrix for a Dirac system.
January 30: Discussion of $\Lambda(V)$ as a functor on the Q -category of finite dimensional spaces V .
February 4: On Bloch's regulator map $K_2(X) \rightarrow H^1(X, \mathbb{C})$. Deligne's approach.
February 5: The torsor associated to a pair of holomorphic functions on a Riemann surface.
February 21, 22: The Japanese τ -function and how the Baker-Akhiezer function can be expressed as a quotient of two τ -functions.
February 22, 23, 24, 25: Baker-Akhiezer function for an elliptic curve.
February 25: Differentials of the first, second and third kinds. Perverse sheaves and holonomic models over a Riemann surface.
February 26: Comment on scattering for $[\partial^2 + q]u = k^2u$. Analogies between gauge groups, holomorphic structures and connections.
February 27: Determinants and line bundles associated to complex structures on a Riemann surface. Singer's determinants and analytic torsion. Calculation on an elliptic curve.

1982-3

- February 28: Poisson summation formula and analytic torsion.
March 1: Holomorphic structures on a vector bundle over an Riemann surface.
March 2: Canonical line bundle over the space of Fredholm maps between two vector spaces.
March 3: The τ -function for line bundles over a Riemann surface.
March 4: Fredholm correspondances.
March 5: Canonical connection on $\mathcal{O}(-1)$ over $\mathbb{P}(V)$.

March 6: Families of vector bundles and families of holomorphic structures on a Riemann surface.

March 8: Determinant of $\bar{\partial} + A$ and Schwinger's paper V .

March 10: ζ -functions of differential operators.

March 11, 12: Determinants and holomorphic structures on a vector bundle over a Riemann surface.

March 13, 14: Clutching functions and projective action of the loop group on the cohomology determinant line bundles when a loop separates a Riemann surface.

March 15: More on the clutching function construction. Explicit Japanese formulas.

March 16: More on vector bundles over Riemann surfaces. Symplectic structure on the loop group ΩG .

March 17: Symplectic structure on G -bundles over an annulus. More on the Kyoto τ -function associated to the Riemann-Hilbert problem.

March 18: Structure of orbits of the automorphism group of a Hermitian bundle over a Riemann surface (Atiyah-Bott situation).

March 19: Instantons.

1982-4

March 20: Feynmann's inequality and Atiyah-Bott convexity.

March 23: Canonical line bundles over $\text{Pic}(X)$.

March 24: Line bundles on \mathbb{C}/Γ .

March 25: On Corrigan, Goddard, Osborn and Templeton: Jet function regularization and multi-instanton determinants. Nucl.Physics B 159 (469-496). Holomorphic structures on the trivial line bundle over \mathbb{C}/Γ and Singer torsion.

March 26: Heat equation methods. Asymptotic expansion of the heat kernel as $t \downarrow 0$.

March 28: Variation of the ζ -function for D^*D with $D = \bar{\partial} + w$ acting on the trivial line bundle over \mathbb{C}/Γ .

March 29: Variation of $\zeta'_A(0)$ where $A = D^*D$, $D = \partial_{\bar{z}} + \alpha$ over \mathbb{C}/Γ .

March 30: Homogeneous distributions and their Fourier transforms on \mathbb{R}^n . Construction \mathcal{D}^{-1} where $\mathcal{D} = \frac{\partial}{\partial \bar{z}} + \alpha$. Green's function on \mathbb{C}/Γ .

March 31: Comology determinant line bundle on a Riemann surface.

April 2: Laplacian and Green's function on a Riemann surface. Seeley's methods for obtaining the asymptotic expansion of a heat kernel.

April 3: $\zeta_{\bar{\partial}^* \bar{\partial}}(0)$ and $\zeta_{\bar{\partial} \bar{\partial}^*}(0)$ (on a Riemannian surface).

April 4, 5: Asymptotics of the heat kernel.

April 6: Formulas of Riemannian geometry.

April 7: Patodi's formulas in the case of Riemann surface.

April 8: Review of WKB. Curvature of some Riemann surfaces.

1982-5

April 9: Finish off computing the heat kernel terms on a Riemann surface. Calculations tested on S^2 . Putting a metric on a line bundle using analytic torsion.

April 12, 13: Continuation of putting a metric on a line bundle using analytic torsion.

April 14: Explicit calculation of ζ for line bundles on \mathbb{C}/Γ .

April 16, 18: Theory in the situation where M is a symplectic manifold and $f : M \rightarrow \mathbb{R}$ is such that X_f is periodic. Atiyah-Bott on Duistermaat-Heckman.

April 19, 20: Parametres for the $\bar{\partial}$ operator over a Riemann surface.

April 23: Atiyah-Singer L^2 index theorem.

April 25: Review of Atiyah's work on K -homology with a view to understanding Connes's classes. Tate's approach to residues. Hochschild cohomology.

April 28, 29, 30: Connes's cohomology.

May 10, 11, 12: The determinant line bundle on the space of holomorphic structure in a vector bundle over a Riemann surface.

May 21: Variation of analytic torsion and calculations involving the heat kernel.

1982-6

May 22: Calculation of regularized trace by means of the heat kernel. Calculation of curvature.

May 23, 24 27: Review calculation of $\langle y | \mathcal{D}^{-1} | y \rangle$ where \mathcal{D} is the $\bar{\partial}$ operator.

May 30: Calculation with the line bundle $\mathcal{O}(P)$ over an elliptic curve.

June 1: Direct ways to get a metric on the dual determinant line bundle over the space of holomorphic structures.

June 3, 4: Calculations on an elliptic curve. Varying holomorphic structures on a Riemann surface.

June 5: Dirac equation: physical derivation.

June 6, 7: Dirac operator on a Riemannian manifold. Calculation for a Riemann surface.

June 7, 8: Adler anomaly: calculating $[D, J]$ where $J(z) = FP \langle z | D^{-1} | z \rangle$. Connection with index theorem.

June 10: Calculations for the case where D has index 1.

1982-7

June 11, 13, 14, 15: Varying the holomorphic structure on the manifold.

June 19, 20: Atiyah's idea. General anomaly formula.

June 22: Coleman denominator for the Green's function.

June 25: Grothendieck Riemann Roch. Hodge Index theorem.

June 26: Hodge Index theorem for surfaces.

June 27: Calculating $c_1(f_!L)$ where L is the line bundle corresponding to a divisor of an elliptic curve $f : X \rightarrow \mathbb{Q}$. Riemann Roch theorem for a vector bundle over a family of curves. Intersection theory for curves on an arithmetic surface.

June 28: Review ζ function and K -groups for varieties over \mathbb{F}_q . Grothendieck's proof of the Hodge Index theorem.

June 29: Review of previous work and consideration of how the torsion changes with the metric.

June 30: Discussion of $\Phi(\theta(-p), E)$ as a ratio of volumes for E a vector bundle over Riemann surface. Falting's lecture on the Shafarevich-Parsin conjecture and applications to determinant line bundle work and the Hodge Index theorem.

July 1: Comment on work for the future on varying metrics and holomorphic structures.

July 3, 4: Formulas for vector bundles arising from clutching onto a given holomorphic bundle and for variation of holomorphic structure.

July 5: Dirac operator over a surface, its determinant line bundle over a space of gauge fields and the global determinant.

July 6: Curvature and determinant theorem for a family $D_t = \partial_{\bar{z}} - \alpha_t$, holomorphic in t .

July 12: Computation of the determinant of a degree 0 line bundle over \mathbb{C}/Γ .

July 18: WKB approach to the determinant term in the heat kernel. Flat Green's functions.

July 30, 31: Summary of some work done in Oxford on denominator free Green's functions.

1982-8

August 6 - 12: Review of calculation of $\det(\partial_{\bar{z}} + \alpha)$ over an elliptic curve. Ground energy as a determinant.

August 12: Graeme Segal's notes on ground energy.

August 14: Differential form version of GRR where fibres are curves.

August 15: GRR applied to families of holomorphic structures.

August 17, 18: Parametrices for $\bar{\partial}$. Introducing flat Green's functions.

August 19, 20, 21: Flat Green's functions.

August 22: Comment on a parametrix for $\bar{\partial}$ and the WKB approximation for the heat kernel e^{-D^*D} .

August 23, 24 25: Anomaly formulas for a denominator-free Green's function.

August 26-30: Discussion of $\langle 0|S|0 \rangle$ as a determinant and related calculations.

September 2 Comment on a Hamiltonian for a 1-particle space.

1982-9

September 4,5: Notes on $\langle 0|S|0 \rangle$. More on GRR for a vector bundle over a family of curves.

September 6,7: On the ζ -function $\text{Tr}(\Delta_y^{-s})$ associated to a parametrized family of curves.

September 10, 11, 12: Equivariant line bundles with equivariant connections and moment maps.

September 13: Review of results from Riemannian geometry.

September 14: Moment map for the action of volume preserving vector fields on metrics of a given volume.

September 15: Complex structures on and oriented plane and points of UHP.

September 16: Continue review of results from Riemannian geometry. Return to complex structures on a surface of fixed volume. Curvature of determinant line bundle.

September 17: Curvature of the tangent bundles along the fibres for a holomorphic family of curves.

September 18, 19: Holomorphic structures on a vector bundle.

1982-10

September 21-27: Holomorphic structures on a vector bundle.

September 28- October 3: Determinants of operators over the one dimensional spaces: S^1 , $[0, L]$ and \mathbb{R} .

October 5: Comment on eigenspaces of the Laplacian.

October 6: Defining $\text{ch}f_!$ as a differential form. Review of perturbation theory. Grassmannian connection.

October 7: On the η -invariant and relationship with determinant.

October 8: Failure of the heat kernel trace to have an asymptotic expansion.

1982-11

October 9: Discussion of the question of existence of an asymptotic expansion in powers of t for $\int e^{-t|x|^2} f(x) dx$ where f is a distribution. Application to the η -invariant. The η -invariant for a manifold with boundary.

October 10: Examples of the η -invariant for manifolds with boundary.

October 11: Boundary condition for the $\bar{\partial}$ operator on a Riemann surface with boundary. Relationship with the Schrödinger equation. Entropy and energy.

October 13: The $\bar{\partial}$ operator as an imaginary time Schrödinger equation with a time dependent Hamiltonian. Certain subspaces of Fock space as boundary conditions for the $\bar{\partial}$ operator.

October 16, 17, 18: Unanswered questions connected with determinants of $\bar{\partial}$ on closed Riemann surfaces. Schwinger's gauge invariant determinant. Results for line bundles over a Riemann surface of degree $g - 1$.

October 21: Continuation of discussion of the $\bar{\partial}$ operator on a vector bundle over Riemann surface with boundary.. Review of τ -function. George Wilson's formula.

October 23, 24: On the τ -function. The τ -function as determinant.

October 27: The KdV equation.

1982-12

October 28: Regularization processes. The algebra behind parametrices and determinants.

October 29, 31: Seeley's and Hörmander's representation of the heat kernel. Determinant formalism.

November 1: Heat kernel e^{-tA} when A is the Laplacian plus lower order terms.

November 2: On Fourier integral operators.

November 3: On $\text{Tr}(e^{itA^{\frac{1}{2}}})$ where A is a positive second order elliptic operator.

November 4: Hörmander's theory for $e^{iA^{\frac{1}{2}}}$ where A is a positive second order operator.

November 5: On an asymptotic expansion for

$$\zeta_A(s) = \text{Tr}(A^{-s}) = \frac{1}{\Gamma(2s)} \int_0^\infty \text{Tr}(e^{-tA^{\frac{1}{2}}}) t^{2s} \frac{dy}{t}.$$

On $\text{Tr}(e^{itP(D)})$ where $P(D)$ is a constant coefficient operator over $\mathbb{R}^n/L\mathbb{Z}^n$. On Hörmander's approach to $\langle x|e^{-itA^{\frac{1}{2}}}|x' \rangle$.

November 6: On $\text{Tr}(e^{-tA}B)$ as $t \downarrow 0$. More Hörmander theory.

November 7: Hörmander's representation of $\langle x|e^{-itP(D)}|x' \rangle$ using asymptotic eigenfunctions of $P(D)$. Summary of wave equation approach.

November 8: Asymptotic expansion as $t \downarrow 0$ of $\text{Tr}(e^{-tA}B)$ where A is a Laplacian and B is a pseudodifferential operator.

November 9: Homogeneous distributions. Remark on $\eta_A(s)$.

November 10: Seeley's method for calculating the asymptotic expansion of the heat kernel $\langle x|e^{-tA}|x'\rangle$.

November 11: Generalities about pseudodifferential operators. Heat kernel e^{-tA} . Comments on Seeley's approach.

November 12: On $\langle x|\frac{1}{\lambda-A}|\xi\rangle$ and $\langle x|A^{-s}|x'\rangle$.

November 14: Pairing elliptic operators and vector bundles.

November 16: Heat kernel formulas and an expression for the anomaly. An expression for the current J_D where D is a Dirac operator and $\text{Tr}((D^*D)^{-s}D^{-1}\delta D)|_{s=0} = \int \text{tr}(J_D\delta D)$

November 17: Proof of the anomaly formula. Relationship between the curvature of the determinant line bundle and the anomaly formula given via the moment map.

November 18: Review Duistermaat's theorem.

November 19: Discussion of the possibility of a Hamiltonian formulation of field theory.

November 20: Local formulas on \mathbb{R}^n for the Dirac operator with coefficients in a vector bundle.

November 22: Notes on Atiyah's talk on Witten's mod 2 anomaly.

November 23: Analytic torsion metric and curvature form arising from holomorphic structures on a given vector bundles over a Kähler manifold.

1982-13

November 22: On Loday's conjecture concerning the homology of $\mathfrak{gl}_n(A)$.

November 23: Weitzenböck formula.

November 25: Letter to Loday about $GL_n(A)$. Question to Atiyah about a vector bundle over a fibre bundle with flat connections along the fibres. Bott's suggestion that Connes's homology is related to the free loop space.

November 26: Preparation for Atiyah's lectures on Witten's ideas for $\text{Map}(S^1, M)$. On operator's with symbol $|\xi|^2$.

November 30, December 1: Second order operators and connections on a vector bundle. The kernel $\langle x|e^{-tD}|y\rangle$ calculated as a path integral.

December 8: Local index theorem for the Dirac operator.

December 10: On Kirwan's Morse theory approach to moduli problems

December 11: Comment on a derivation of a path integral expression for the heat kernel. Kolmogorov-Chapman type differential equations. Local index formula for the Dirac operator over an even dimensional spin manifold.

December 12: Path integral approach to a local index formula.

December 13: Heat flow on a Riemannian manifold.

December 14: Grassmannian calculations.

December 17: On Witten's talk on the equations of motion for the σ -model. Atiyah's formulation.

December 18. Path integrals and random walks on manifolds.

December 20: Supersymmetry.

December 21: On families of $\bar{\partial}$ operators on a vector bundle over a Riemann surface. Account of a conversation with Donaldson.

December 22: Equivariant cohomology of the gauge group acting on the space of connections, calculated using differential forms. Preparation for a talk with G. Wilson.

December 23, 25: Equivariant cohomology using differential forms.

December 26: To find a Lie analogue of $H_{\mathcal{G}}^*(\mathcal{A}) = H^*(B\mathcal{G})$ where \mathcal{G} is a gauge group and \mathcal{A} the space of connections.

December 27: Relating $e_2 \in H^3(U_n)$ to the central extension of ΩU_n .

December 28: Equivariant curvature.

December 29: Existence of an odd form $\text{ch}(f)$ when $f : E \rightarrow F$ is an isomorphism of vector bundles.

December 30: Ideas suggested by Connes's formulation of the index theorem.

1982-14

Notes on a lecture of Serre on the number of points on a curve.

Contents 1983

1982-1

- December 31, 1982 Definition of a Connes cocycle.
- January 1,2: On Connes theory. Discussion of the goal to find the character, expressed as an equivariant form, of the index of a family of operators parametrized by a space of connections.
- January 3: Characteristic classes for flat bundles.
- January 6,7: Equivariant De Rham cohomology for the gauge group \mathcal{G} .
- January 8: Comment on regularizing Green's functions.
- January 9,10: Characteristic classes for the index of virtual bundles. Characteristic classes on a Grassmannian and links with Connes theory.
- January 11: Representations of the symmetric group.
- January 12: Connes homology for $A = k \oplus \overline{A}$ where $\overline{A}^2 = 0$.
- January 13: Calculation of Connes homology and Hochschild homology for Lie algebras.
- January 14: Letter to Loday on maps between the Connes groups and Lie K -groups.
- January 16: Discussion of Witten's work with the operator $d + si(X)$ and connections between the Connes complex and the loop group.
- January 18: Invariant differential forms on a gauge group.
- January 19: Connes's basic examples of cocycles. Lie theory versus the discrete group.
- January 20, 21: Preparation for lecture on the determinant line bundle. Elliptic curve example.
- January 23: Determinant line bundle and singular set. New idea for producing invariant forms.
- January 24: Brief discussion on using ideas of Atiyah and Bott to produce equivariant forms in the presence of a circle action.
- January 25: Computations with the free loop groups of U_n . Discussion of aim to define a character for the index of a family of Dirac operators.

1983-2

- January 27: Formulas relevant for calculating the equivariant form.
- January 28: Preparation for lecture on the determinant line bundle including the use of the formula $\text{Tr}_{(\text{reg})}(D^{-1}\delta D)$.
- January 29: Transgression in the Chern-Simons paper applied to GL_n - bundles.
- January 30: Relating Lie K -theory to algebraic K -theory. Characteristic classes of representations and implications for de Rham cohomology, Connes cohomology and Deligne cohomology.
- February 1: On a paper of Benora, Cotta-Ramasino: Remarks on BRS Anomalies and Gauge Transformation Groups.
- February 2: Constructing classes in $H^*(B\mathcal{G}_\delta)$ where \mathcal{G}_δ is the discrete group underlying \mathcal{G} .
- February 4: Preparation for the third lecture on the determinant line bundle and its connection.
- February 16, 17: Continuation of work on defining the character of the index as a differential form, metric version.
- February 18, 19: Computing the differential of $\text{Tr}_{(\text{reg})}(D^{-1}\delta D)$ for the Riemann surface case.

February 20, 22: Circle case: $\mathcal{G} = \text{Maps}(S^1, U_n)$ and \mathcal{A} , the unitary connections.

February 23: Meaning of an analytic proof of an index theorem for families.

February 24: Comment on problems encountered with current approach.

February 25: Preparation for lecture 5: ζ -function determinants or analytic torsion.

February 26: More on the problem of finding an analytic formula for the index of a family of operators.

February 27: Formal structure of proof of the index theorem.

February 28: Idea from Bott-Chern paper.

March 1: Bott-Chern formulas in the holomorphic setting.

March 2: Curvature calculations for the determinant line bundle belonging to a family of $\bar{\partial}$ operators over a Riemann surface.

March 3: Generalizing $\text{Tr}(e^{-tD^*D}) = \text{Tr}(e^{-tDD^*})$. Review Schwinger calculation using Witten formulas.

March 6: Preparation for 7th lecture: Computation of the curvature for \mathcal{L} using the analytic torsion metric.

March 5: Bott-Chern theory applied to investigating:

$$[ch(E) - ch(F)] - [ch(Ker(D)) - ch(Coker(D))] = d(???).$$

March 6: Eigenvalue calculations for variations of D^*D .

March 6, 9: Discussion of the problem with finding an analytic proof of an index theorem for a family of Dirac operators. Review of spinors, Clifford algebras and their K -theory, Dyer-Kan classification theorem for diagrams of simplicial sets.

1983-3

March 10, 11: Calculations for a family of Dirac operators on $\mathbb{R}^p/\mathbb{Z}^p \times \mathbb{R}^q/\mathbb{Z}^q$.

March 11: Notes for 7th lecture.

March 12: Clifford algebras and Dirac operators.

March 13: Dirac operator on $(S^1)^{n-1} \times S^1$, $n = 2m$.

March 14, 15: Kasparov cup product. Further work on families of Dirac operators.

March 16: Lagrangian viewpoint.

March 17: Path integral approach. Heat kernel over a product of two tori. Path integral formula for the Dirac operator over Euclidean space.

March 18, 20, 23: Calculating diagonal values for the kernel $e^{-t\Box}$.

March 24: Family of Dirac operators and the Connes algebra assigned to a foliation.

March 27: Calculation of the index of the standard harmonic oscillator $d + d^*$ on \mathbb{R}^n . Comments on the Patodi approach and the Seeley approach to the asymptotic expansion of $e^{-t\Box}$.

March 28, 29: Physics approach to calculating terms in the heat kernel.

March 30: Further calculations for the heat kernel.

March 31: Review $\zeta(-k)$, $k = 0, 1, 2, \dots$ and the Adams operations in K -theory.

April 1: Arekelov-Faltings intersection theory on arithmetic surfaces. Related questions and ideas.

April 3: Return to holomorphic vector bundles over Riemann surfaces.

April 4: Calculating constant term in $\text{Tr}(e^{tD^*D}D^{-1}B)$ as $t \rightarrow 0$.

1983-4

April 8, 9: The fermion C^* algebra.

April 10, 11: A C^* algebra and its K -theory, particularly the Kronecker foliation algebra.

April 12: Cross products and factors in C^* algebras. Atiyah's L^2 index theorem.

April 13: Asymptotic behaviour of $\text{Tr}(e^{-tD^*D}D^{-1}\delta D)$ as $t \downarrow 0$ where $D = \bar{\partial}$ on a Riemann surface.

April 14: The determinant line bundle over a space of constant coefficient $\bar{\partial}$ -operators on a trivial line bundles over a 2-dimensional torus.

April 16, 17, 18: Review the Ray-Singer calculation of torsion on elliptic curves.

April 19: The GRR formula for a family of constant coefficient $\bar{\partial}$ -operators over a torus.

April 20, 22: Connes K -theory of a foliation, especially the Kronecker foliation.

April 23: Identifying $\mathcal{S}(\mathbb{R})$ equipped with the operators translation by 1 and multiplication by $e^{2\pi i x}$ with sections of the trivial line bundle over \mathbb{T}^2 .

April 24: Calculating \mathbf{Pic} of the orbit topos of the Kronecker foliation. Review of θ -functions.

April 25, 26: A construction using von Neumann algebras of type II which shows that a flat unitary bundle gives zero in $K(X) \otimes \mathbb{R}$.

April 27, 28: Principal bundles with discrete structure group.

April 29, 30: Equivalence of holomorphic structures and metrics on a compact oriented surface of fixed volume.

May 1: Conversation with George Wilson about the Yang-Baxter identity.

May 7: Preparation for talk on Arakelov-Faltings theory and zeta determinants.

May 8: Computing analytic torsion for line bundles. Results from Falting's paper.

May 9: Talk on Arakelov-Faltings theory.

1983-5

May 10, 11: Discussion of the possibility of a K -theory of holomorphic vector bundles.

May 12: K -theory of non-unital rings.

May 18: Preparation for K -theory conference in Luminy.

May 19: Return to discussing families of holomorphic curves, seeking inspiration from the work of Connes work and Feigin- Tsygan.

May 20, 21, 22, 23: Calculations inspired by the Connes results on the Kronecker foliation.

May 25: Notes on cyclic homology. Mixed Hodge structure for a non-singular variety which is not complete. Removing points and discs from a Riemann surface.

May 26: The KMS condition.

may 27: Cyclic homology calculation- forward shift.

May 29: Why $\prod_{n=1}^{\infty}(1 - q^n)$ is a modular function.

May 30: Cyclic cohomology and cyclic graphs.

June 1: On cyclic homology for rings without unit.

June 2: Relative cyclic homology of A modulo k .

June 3: Relating the Quillen approach to cyclic homology from Hochschild homology to the Connes approach through the non-commutative de Rham complex. Connes definition as a functor on cyclically ordered finite sets.

June 4: The cyclic category.

June 5: Hsiang and Staffeldt result that $HC_p(T(V), k) = 0$ if $p \neq 1$. Comparison with de Rham homology.

June 6: Cyclic and de Rham homology.

June 9: Preparation for a paper on cyclic homology giving an exposition of some aspects using the double complex.

1983-6

June 30- July 4: Review of progress on the index theorem for families.

July 5: Summary of analytic progress and change of direction to a geometrical attack. Introduction of form $\text{Tr}(e^{-tL^2 + \sqrt{t}dL + \Omega})$.

July 6: Summary of progress on ideas on the cohomology of gauge groups.

July 7: Invariant forms on G give natural transformations from flat connections on $Y \times G/Y$ to $\Omega^*(Y)$.

July 8, 9: Review of Bott-Chern formulas and applications to flat bundles on the trivial principal \mathcal{G} -bundle over Y .

July 10: Calculation of characteristic classes using Maurer-Cartan form.

July 14: Using the transgression formula $\int_0^1 dt(e^{td''w + (t^2 - t)w^2})$.

July 15: Preparation for letter to Loday on the natural transformation from the Connes complex $\mathcal{C}(A)$ to the filtered de Rham complex based on using $\int_0^1 dt(e^{td''w + (t^2 - t)w^2})$.

July 17, 18: Connes periodicity operator. Connes index: $\text{Index}(epe) = \text{tr}((p^{-1}[p, e])^{2m+1})$.

July 19: Loday's conjecture on the filtration of cyclic homology obtained from \mathfrak{gl}_n for different n .

July 20: Defining homology classes using the Chern-Weil curvature process for $\tilde{\mathfrak{g}} = \mathfrak{gl}_n(A)$ with values in the filtered de Rham complex.

July 21, 22: More work on Loday's conjecture. Letter to Loday.

July 23, 24: Fadeev-Popov Ansatz. List of ideas and problems. Formal category of a scheme and related de Rham complexes.

July 25: Review of Feynmann diagrams, effective potentials and vertex functions.

July 26: Review normalization (Lee model).

July 28, 29, 30, August 1: Characteristic classes for $H^*(\mathcal{G})$. How to realize $\text{ch}(E_{\text{invar}})$ on $H^*(B\mathcal{G} \times M)$ by equivariant forms on $\mathcal{G} \backslash \mathcal{A}$.

1983-7

August 7: Determining $H^*(B\mathcal{G})$ and realising primitive generators of $S(\mathfrak{g}^*)$ as differential forms. Contrast between compact group and gauge group cases.

August 8: Comment on continuous cohomology. Is $H_c^*(\mathcal{G}, \mathcal{M}) = H^*((\mathcal{M} \times \Omega^*(\mathcal{A})^{\mathcal{G}}))$?

August 9: More work on $B(\mathcal{G})$ and $H^*(\mathcal{G})$.

August 10, 11, 12: The Lie algebra of the gauge group and Gelfand-Fuks cohomology. The map $H^*(\mathcal{G}) \rightarrow H^*(\mathcal{G})$. Rational cohomology of $B\mathcal{G}$ and \mathcal{G} .

August 13: Continuation of the program to determine the continuous and Lie algebra cohomology of gauge groups. Conjecture on primitive generators of the cohomology of \mathcal{G} , \mathcal{G} , $B_c\mathcal{G}$, and $B\mathcal{G}$.

August 17: On the Polyakov formula for $\det(\not{D} + \not{A})$ on \mathbb{R}^2 .

August 19: On normalization.

August 20: Feynmann's formula for $\frac{1}{ab} = \int_0^1 dt \frac{1}{[ta+(1-t)b]^2}$. Field theory of a real-valued function $\phi(x)$, $x \in \mathbb{R}^n$ given by action $S(\phi) = \int d^n x \{ \frac{1}{2} \phi(-\Delta + m^2) \phi + \frac{\lambda}{4!} \phi^4 \}$.

1983-8

August 21: Motion of a particle on the line governed by the Hamiltonian (anharmonic oscillator) $H = \frac{p^2}{2} - \frac{w_0^2}{2} x^2 + \frac{\lambda}{4!} x^4$.

August 29, 30: Magnetism.

September 1: Conversation with Jackiw on anomalies and σ -model approximation to low energy QCD.

September 9: BRS and Dixon's work.

September 10: Review problem left over from Loday letter. Review determinant line bundle.

September 15: Discussion of whether there is a direct connection between cyclic cohomology and anomalies.

September 17, 18: Discussion of Connes Λ -interpretation of cyclic cohomology and the discussion of compatibility of two maps from the Lie algebra homology to Deligne cohomology.

1983-9

September 20, 22: Construction of character forms associated to an invariant connection on an equivariant bundle in equivariant cohomology.

September 24-29: Determining $H^*(B\mathcal{G})$.

October 2: Lifting a $\mathfrak{gl}_n(A)$ cycle with values in the De Rham complex to one with values in the complex $\mathcal{B}(A)$. Amitsur complex.

October 4: Observations from a paper of Witten on baryons.

1983-10

October 9: What is $\text{Ext}_\lambda^*(k^\natural, A^\natural)$? Return to problem raised in letter to Loday on constructing a cocycle for $\mathfrak{gl}_n(A)$ with values in the double complex $\mathcal{C}(A)$.

October 10: Chain complexes $\Omega^*(Y, P \times^G V)$ and $C^*(\mathfrak{g}, V)$, and connection with Lie algebra cohomology. Formulas for the boundary operators b and B in $\mathcal{C}(A)$.

October 11: Karoubi's non-commutative differential algebra of forms $\overline{\Omega}(A) = \Omega(A)/[\ , \]$. Application of Connes theorem to the Loday problem.

October 13, 14: Chern characteristic classes of $\mathfrak{gl}_n(A)$ with values in non-commutative de Rham cohomology of A .

October 17, 18: Curvature of the Grassmannian connection form $\text{ch}_n = \frac{1}{n!} \text{tr}(e(de)^{2n})$. Index formula for F : $\text{Index} = (-1)^n \text{tr}(\epsilon e[F, e]^{2n})$.

1983-11

October 20: Maps $K_0(A) \rightarrow HC_{2n}(A)$ and $K_1(A) \rightarrow HC_{2n-1}(A)$.

October 22, 23: Proving $\mathcal{B}(A)_{\text{red}}$ is quasi-isomorphic to $\overline{A}^{*(+1)}/(1+t, b)$. Lie algebra cohomology of the gauge group and how it is related to the index and determinant questions.

October 26, 27: Connes S -operator.

October 29, 30,: Trace for 1-summable Fredholm modlues.

October 31, November 1: Connes S -operator.

November 2: Connes-Karoubi theorem: $H^p(\overline{\Omega}) = \text{Im}\{S : \overline{HC}_{p+2}(A) \rightarrow \overline{HC}_p(A)\}$.

1983-12

November 10: Computing transgression $H^*(B\mathcal{G}) \rightarrow H^*(\mathcal{G})$. $(BG)^{S^1} \equiv PG \times^G (G_c)$ where G_c denotes the G space with G acting as conjugation.

November 11: Discussion of Singer's approach to calculating transgression.

November 12, 13: Calculating transgression using the Chern-Simons form.

November 14, 15: Transgression formula: $\text{tr}(e^{F_A}) - \text{tr}(e^{F_B}) = d \int_0^1 dt \text{tr}(A-B)e^{(1-t)F_B + tF_A - t(1-t)(A-B)^2}$.

November 16: Notes on an anomaly formula: $c_1(\overline{\mathcal{L}}) = \int_{M^{2n}} (\text{ch}(E)\hat{A}(M))_{n+1}$. Connections on the principal bundle.

November 17: Calculations on the principal bundle. Equivariant curvature in both D and A notations.

November 18: Constructing characterisitic classes for \mathcal{G} -bundles out of character classes for U -bundles.

November 19, 20: Formulas for gauge transformations on the principal bundle. Action of gauge transformations on connections: $g^{-1} \circ D \circ g$ on $\Omega(M, E) \leftrightarrow d + g^*A$ on $\Omega(P) \times V$. Transgression map: $W(\mathfrak{g})_{\text{basic}} \rightarrow \Lambda(\mathfrak{g})^G$.

November 21: Check transgression calculations.

1983-13

November 21: Letter to Singer on transgression formulas. Checking left and right actions of the gauge group.

November 26: Review of local index formulas. Some new ideas.

November 27: Review construction of characterisitic classes for the Lie algebra of gauge tranformations as in letter to Loday.

November 28: Constructing invariant forms on \mathcal{G} .

November 29: On Atiyah's suggestion that Quillen's formula associated with $e^{tL^2 + \sqrt{t}[D, L] + F} = e^{tL^2 + \sqrt{t}d^\mu[D_\mu, L] + \frac{1}{2}dx^\mu dx^\nu F_{\mu\nu}}$ and Getzler's proof should be part of the same framework.

November 30, December 1,2: Comment on Wess-Zumina Lagrangian as described by Witten. Preparation for letter on transgression.

December 3: Comment on Singer's intention on using his vol_B construction and how it works for flat connections.

December 4: Summary of formulas and explanation of apparent paradox.

1983-14

December 6: Talk in Jaffe's seminar. Notes of a conversation with Luis and Ginzpang about anomaly formulas.

December 8: Describing anomalies using cyclic cohomology.

December 11: Note that Witten-Alvarez obtain the \hat{A} -genus by means of a constant EM-field. Fermion quantum mechanics.

December 13: Lot's problem.

December 14: Fujikawa's approach to anomalies.

December 15: Return to Lot's problem. Witten's table and QCD.

December 16: Facts about QCD. Witten's observation about a physical interpretation of stable homotopy groups.

December 17: On the Connes S -operator.

December 18: The local index theorem using path integrals on Euclidean space with arbitrary gauge potential. Digression on spinor representations in $2n$ and almost complex manifolds.

December 19: The index of the Rarita-Schwinger operator.

December 21: Discussion of Clifford algebras.

December 22: Weyl quantization and its fermion version.

December 23: Developing a formal theory of path integral and fermion integration theory.

December 24: Quantizing the Toeplitz process. Fermionic analogues.

December 28: Review transgression process for constructing differential forms on \mathcal{G} .

December 29: Local index formula for a family of Dirac operators on M parametrized by a family of connections quotiented by a gauge group.

1983-15

December 30: Is the cohomology class of the form $\text{tr}_E(\epsilon_E e^{L^2 + [D, L] + D^2})$ independent of L ?

December 31: Super-trace.

1983-Calculations relevant to cyclic theory

1983-Ideas

1983-Lecture Notes 0

Quillen: First lecture on local index theory

1983-Lecture Notes 1

Donaldson: Generalization of a theorem of Narasimhan-Seshadri

1983-Lecture Notes 2

Getzler: New proof close to Kotake.

1983-Lecture Notes 3

Kirwan: Theorems about convex bodies.

1983-Lecture Notes 4

Atiyah: Integrals over fixpoint submanifolds.

1983-Lecture Notes 5

Quillen: Review of local index theorem.

1983-Lecture Notes 6

Ginsparg: Obtaining the Chern-Simons form.

1983-Lecture Notes 7

Witten: Equivariant index theorem.

1983-Lecture Notes 8

Witten: Some QCD inequalities.

1983-Lecture Notes 9:

Witten: Tables.

1983-Lecture Notes 10

Getzler: Heat kernel of $H = \frac{1}{2}[-\Delta + A^*A]$.

1983-Lecture Notes 11:

Kazhdan and Thomas Parker: On Hecke algebra $C_c^\infty(G)$. Super-symmetric σ -model.

1983-Lecture Notes 12

Atiyah: Newton Polyhedra and Algebraic Geometry.

1983-Lecture Notes 13

Cuntz and Connes: Quasi-homomorphisms. Cyclic cohomology.

1983-Lecture Notes 14

Schroer: Super-symmetric theory

1983-Lecture Notes 15

Patterson: New results in ergodic theory.

1983-Notes

On cyclic theory.

1983-Review

Of Getzler material.

Contents 1984

1984-1

- January 1,2: Super-connections and super-traces.
- January 4,5: Super-groups and super-Lie algebras.
- January 6,7: Super-manifolds.
- January 8: Odd K -elements.
- January 9: Claims for the form $\text{tr}_E(\epsilon_E e^{D^2+[D,L]+L^2})$ where L is invertible.

1984-2

- January 10: A new proof that the Chern character cohomology class is independent of the choice of connection.
- January 11: Checking formulas of January 2.
- January 13: Some Banach algebra formulas.
- January 15, 16, 17: Odd K - classes. Notes on Clifford modules and super-vector bundles.
- January 18: Notes on a wide ranging conversation with Connes.
- January 19: The Hodge $*$ operator and Clifford multiplication. Comments on the Novikov conjecture.
- January 24: Description of various notational conventions.
- January 25: Comment on K -theory of Clifford algebra bundles.
- January 27: Discussion of local Index formula for a family of Dirac operators with coefficients in a vector bundle.
- January 28, 29: Bott's theorem on Chern numbers. Witten's cohomology identified with localized equivariant cohomology.
- January 30: Notes on a conversation with Atiyah on Frankel's observation about a circle action on a symplectic manifold. Kirwan's work. Donaldson's proof that stable bundles have unique YM connections. Novikov Conjecture. Conversation with J. Roe.
- January 31: Notes on a conversation with Scott Petrach on Duistermaat-Heckman type proof of the Index theorem. Atiyah's summary of a talk by Berry.
- February 1: Conversation with Connes. Cyclic theory for a ring with no unit.
- February 2: Account of conversation with Katz, Gabber and Deligne.
- February 3: Conversation with Connes on $H^*(\Gamma) \rightarrow H_\lambda^*(\mathbb{C}[\Gamma])$, wave packet transform and Novikov conjecture.
- February 4: Connes method for extending cocycles from A to $\text{End}_A(E)$ where E is a projective module over A .

1984-3

- February 5: Morita invariance of cyclic cohomology.
- February 6,7: Connes construction to kill curvature.
- February 8: Connes proof of Morita invariance.
- February 9: Connes approach by tensoring an elliptic operator with a vector bundle. Relating cyclic cocycles and heat kernels.

February 10: Characterizing Dirac operators.

February 11: Constructing a cyclic cocycle belonging to a Dirac operator.

February 12: Constructing cyclic cocycles representing the Chern character of a Dirac operator.. K-theory of a super-algebra.

February 17: Kasparov K -theory of graded C_k -modules.

1984-4

February 18: Check the Index theorem (using a heat kernel method) over a torus for the Dirac operator, $-i\mathcal{D} = -i\gamma^\mu D_\mu + \epsilon L$ acting on $S \otimes E$ where L is a selfadjoint operator.

February 19: Formula for the wave packet formula in Euclidean space. Bott's idea related to his work on the Lefschetz fixed point theorem.

February 20: On a rigorous proof of the classical limit formula $\text{Tr}(e^{-\beta H}) \sim \int \left(\frac{dpdq}{2\pi h}\right)^n e^{-\beta H}$ where $H = \frac{p^2}{2} + V(q)$ is quantized as the operator $-\hbar^2 \nabla^2 + V(x)$. On a proof of the Index theorem using the idea that the quantum partition function has a classical limit which is an integral over the cotangent bundle.

February 21: On Connes groupoid and algebra of operators depending on \hbar .

February 22: The Index theorem for the Dirac operator interpreted as a supersymmetry operator. Partition function, Clifford algebras and Weyl algebra.

February 24: Lecture on Clifford algebras. Connes algebra of operators. Comments of Atiyah on the Index theorem.

February 25: Fundamental class in K -theory.

February 26: Attempt to derive a proof for the Index theorem for the Dirac operator using an isometric embedding.

February 27: Thom isomorphism in K -theory for complex vector bundles.

February 28: Further discussion of the Index theorem for Dirac operators and a generalisation to an assertion about maps.

1984-5

February 29: Computation of the character of the Thom class $i_!1$ in the K -theory of a vector bundles equipped with a metric, connection and Spin^c structure.

March 1: On the Berezin determinant.

March 3: Correspondances between approaches to the Index theorem in algebraic geometry and the Riemann-Roch theorem for complex manifolds.

March 4: Further discussion of the Index theorem for the Dirac operator.

March 5: Concept of a Dirac operator. Conversation with Graeme Segal.

March 6: Discussion of Riemannian geometry calculations connected with a family of Dirac operators.

March 7,8: Return to the calculation of $\text{ch}(i_!1)$ where $i : M \rightarrow E$ is the zero section of a complex vector bundle.

March 8: On a formula for $\det we^{-\tilde{J}w^{-1}J}$.

March 9: Classical mechanics, quantum mechanics and classical limit formalism.

March 10: Review of the Thom class formula.

March 11: Signature operator and Gaussian-Thom form.

March 12: Discussion on the Dirac operator and $d + d^*$.

1984-6

March 13: Splitting the operator $d + d^*$ on a Kahler manifold. Notes for a lecture on the Chern character form $\text{tr}_s \left(e^{(D+L)^2} \right)$.

March 14: The Thom form on the tangent bundle.

March 15: Conversation with Atiyah on a local Index theorem for connections with non-zero torsion. Convolution algebra of differential forms.

March 16, 17: Discussion of the limiting heat kernel as an n -form on the tangent bundle.

March 20: Index theorem on a torus for the Dirac operator with coefficients in a vector bundle equipped with a connection.

March 21: Limiting heat kernels.

March 23: Review of recent work.

March 24: Classical limit for the heat kernel for the Dirac operator with values in a vector bundle with a connection.

March 25: Discussion of Friedan-Winney paper.

March 26: Fermion Lagrangian quantization.

March 27: When does a quadratic form $S(q, q')$ define a symplectic transformation $(q', p') \mapsto (q, p)$? Hyperbolic quadratic forms and orthogonal transformations.

1984-7

March 28: Path integral approach to the heat operator $e^{-\beta H}$ and the limiting heat operator.

March 29: The super-group $\mathbb{R}^{1,1}$, its convolution algebra, left and right infinitesimal translations: $\partial_\theta \mp \theta \partial_t$.

March 30: Critical points for the fermion Lagrangian.

March 31: Friedan-Winney action. Super-heat operator.

April 1: Further discussion of the limiting heat kernel.

April 2,3, 4: Motion in a uniform magnetic field.

April 6: Dirac operator, general metric, coframe w^m calculation.

1984-8

April 7: Review calculation of April 6. Getzler's filtration. Dirac operator for a constant magnetic field.

April 8: Notes on a conversation with Atiyah in Witten's office.

April 9: Problems with the idea of a classical limit for the super-heat kernel. Parallel transport using a path integral.

April 10: Graded subalgebras of $A \otimes k[h]$ where A is an algebra over k .

April 11: Further thoughts on the local Index theorem.

April 12: The Grassmannian graph construction.

April 15: An extension of the algebra of asymptotic differential algebras.

April 18: Process of going from the cyclic cohomology of $\Omega^0(M)$ to that of $\Omega^0(M, \text{End}(E))$ and restriction $\Omega^0(M) \hookrightarrow \Omega^0(M, \text{End}E)$ with applications to cyclic cohomology.

April 20: Marmonic oscillator heat kernel.

April 21: Implementation of the isomorphism taking the cyclic cohomology of $\Omega^0(M)$ to that of $\Omega^0(M, \text{End}E)$ by the cochain $\int dt \text{tr} \left(e^{D^2 + t[D, \theta] + (t^2 - t)\theta} \right)$.

April 22: Review of November and December's work on transgression. More discussion on a local Index theorem.

1984-9

April 23: Relationship between the Chern character associated with a super-connection and that coming from the Grassmannian graph construction.

April 29: Summary of recent ideas.

April 30: Constructing cocycles from odd derivations and related universal algebra.

May 2: Twisted polynomial rings and a new approach to the heat kernel of the harmonic oscillator.

May 3: Existence of the heat operator e^{tD^2} in the Weyl algebra of operators of the form $\int d\sigma f(v) e^{Dv}$.

May 4,5: Deriving the trace of e^{tD^2} using the path integral. Derivation of the Schrödinger kernel of the harmonic oscillator using the explicit metaplectic formula.

May 6: Weights in a Von Neumann algebra, KMS condition.

May 7: Computing $\text{Tr} (e^{-H} e^{v \cdot D})$ where $H = -D_\mu^2$ and $v \cdot D = v^\mu D_\mu$ using the KMS condition.

May 8: Calculations associated with showing the existence of the heat operator.

May 9: Fundamental solution of the heat operator.

May 10: Construction of the heat operator e^{-tH} where $H = p^2 + V(q)$.

May 11: Hadamard's method of constructing the heat kernel for the Laplacian of a curved manifold.

1984-10

May 12: Volterra's method for constructing e^{-tH} . Discussion of a method of proof of the existence of the heat operator for the harmonic oscillator by deformation of the classical case where $\hbar = 0$.

May 13: The heat operator for the covariant derivation Laplacian with coefficients in a vector bundle.

May 14, 15: Working with pseudo-differential operators in \mathbb{R}^n .

May 16: Review of past few day's work. Attempt to construct e^{-tH} as a kernel on the tangent groupoid of a manifold.

May 17, 18, 19: Attempt to put together a complete proof of the existence of the heat operator depending on Planck's constant, together with an asymptotic evaluation of the trace as $\hbar \rightarrow 0$.

May 20: Review parametrix method and Fredholm theory.

May 21: Review local Index theorem for families.

May 22: Proof of Index theorem for $M = \mathbb{R}^n/\Gamma$.

May 24: On Kasparov K -theory.

1984-11

May 25: Character form and examples coming from projections. Analogue for the plane of the Hilbert operator. Fourier transform of $\frac{1}{|z|^k|z|^{2s}}$.

May 26: Operators in Kasparov's K -theory. Cocycle computation for constant coefficient operators in \mathbb{R}^n .

May 28: Discussion of the general picture for families of Dirac operators.

May 29: Laplace transfor of $\text{tr}_s \left(e^{u(L^2+dL)} \right)$ where L is a family of odd-degree skew adjoint operators endomorphisms of a super-vector space. Grassmannian graph construction.

May 30, 31: Grassmannian graph approach to the local Index theorem. Representation of the k th Chern character form as $\frac{1}{2}\text{tr}_s \left(\frac{1}{\lambda-L^2} dL \right)^{2k}$. Links between the heat kernel and resolvent approaches to the Chern character.

June 2: Transgression problem from the point of view of the Grassmannian graph construction.

1984-12

June 10: Transgressing the character form $\text{ch}_k = \frac{1}{2} \frac{1}{k!} \text{tr}_s \left(\frac{\epsilon}{\epsilon^2-L^2} dL \right)^{2k}$ to the group acting on the Grassmannian graph.

June 11: Abstract discussion of character forms.

June 24: Review of the relationship between character forms defined by the super-connection formalism and that by the Grassmannian graph method, i.e., the link between the heat operator approach and the parametrix approach.

June 25: Review of Witten's approach to the determinant line bundle over a space of connections.

June 26: Baum-Connes problem: to define a good notion of equivariant cohomology.

June 28: Review of transgression situation.

June 29: Transgression calculations. More discussion of the super-connection and the Grassmannian graph formalisms.

June 30: Understanding of the relationship between the super-connection and the Grassmannian graph formalisms. Discussion of the super-character: $\text{tr}_s e^{u(L+[D,L]+D^2)}$

July 1, 2: Work on an attempt at writing paper for Gelfand on the determinant line bundle.

July 7: Description of two interesting problems.

July 9: Link between Connes S -operator and Bott periodicity. Discussion of the bilogarithm and Bott periodicity. Local Index theorem for families - odd degree case.

July 10, 11: Work on local Index theorem for Dirac operators using super-connections. The η -invariant for the Dirac operator over a circle.

1984-13

July 12: Further discussion of the transgression process. Comment on multiplicative K -theory.

July 13: Constructing regulator maps $K_{2n-1}\mathbb{C} \rightarrow \mathbb{C}^\times$ by Karoubi's method using relative K -theory and Deligne cohomology. Discussion of parallel transport from the space of connections on the trivial bundle over S^1 to the unitary group.

July 14: Construction of left invariant character forms on the loop group.

July 15: Dirac operators and Weitzenböck formula using the principal frame bundle.

July 17, 18: Link between super-connections and the ζ -formula for ch_1 of the determinant line bundle. Odd version of Grassmannian graph.

July 17, 18: Transgression for the e_1 class. Bott periodicity from the differential point of view.

July 19: Continuous cohomology and periodicity. Dilogarithm.

July 20: Connes description of the multiplicative map on $K_1^{alg}(A)$ associated to a trace on A . Novikov conjecture and Mischenko's construction.

1984-14

July 21: On Bismut's proof of the Index theorem for the case when $M = \mathbb{R}^n/\Gamma$.

July 24: Ito's equation.

July 25: More on Bismut's paper and Ito's ideas. Review of what we know on the Index theorem and path integrals.

July 26, 27: Physicist's proof of the Index theorem using path integrals and fermion integrals. Comparison with Bismut's approach. Review of the super-group $R^{1,1}$ and Freidan-Winney. Bismut's factorization of parallel transport in $S \otimes E$.

July 28: Review construction of $\langle x_0 | e^{-hD^2} | x_0 \rangle$. Random walk on \mathbb{C}^* : $dz_t = z_t idw_t$, where w_t is Brownian motion.

July 29: Understanding the fermion integral: $\int \mathcal{D}\psi(t) e^{\int (\psi\dot{\psi} - \psi A\psi) dt}$. Idea's behind Bismut's paper. Possible approach to computing the heat kernel in a Weyl algebra.

July 30, 31: Fermion integrals and quantization. Some super-connection identities.

1984-15

August 1: Bismut's form on $\mathcal{L}M$, the loop space of M .

August 2: Interpretation of equivariant cohomology of ΩM and $\Omega BU(k)$. The $\Omega BU(1)$ case - Bismut's form is not an equivariant form but a Witten form.

August 3: Differential geometry of the normal bundle. Comment on Bismut's map

$H(\Omega^*(\Omega M)^{S^1}, d - wi_X) \rightarrow H_{DR}(M)$. Comments on 'The Index Theorem and Equivariant Cohomology of the Loop Group', by J-M Bismut.

August 4: Continuing with attempt at a local Index theorem. Review of Godbillon-Vey class and characteristic class for a codimension n foliation with trivial normal bundle.

August 5: Calculations for a 1-parameter family of connections.

August 6: $\partial_t \not{D}$ for the Dirac operator associated to a 1-parameter family of metrics.

August 7: Transverse connection on $T_{X/Y}$.

August 8: Curvature calculations.

1984-16

August 18: Berline-Vergne proof of the Index theorem.

August 19: The Dirac operator on a Riemann surface.

August 20, 22: Bismut's Witten form on $\mathcal{L}M$.

August 23: Constructions using X/Y , and S^1 bundle over base Y .

August 24: Bismut's form is defined for an S^1 action. Analogies between cyclic theory and equivariant cohomology of the free loop space. Links between $\mathcal{L}M$ and $S^{-1}\text{Ext}_\Lambda(k^\natural, A^\natural)$.

August 29: Summary of recent ideas (generalize Bismut to Kac-Moody).

August 30: Gaussian processes.

August 31: Comment on constructing the heat operator.

September 1: Can the heat operator be constructed via $T\left(e^{\int dt \psi_t \mathcal{D}}\right) = e^{-t\mathcal{D}^2 + \theta D}$?

September 2: Applying Connes-Gezler theory.

September 5: The Clifford algebra as a deformation of the exterior algebra and an attempt at an analogous treatment of the Weyl algebra.

September 6,7: Twisted polynomial algebra. The convolution algebra belonging to the Weyl algebra and formula for e^{-tp^2} .

September 9: Review of standard tensor calculus formulas.

September 17: Singer's suggestion that $\text{tr} T\left(e^{\int (-\dot{x}^\mu A_\mu(x) + \frac{1}{2} \psi^\mu \psi^\nu F_{\mu\nu}(x)) dt}\right)$ is the trace of super-parallel transport.

September 18: Formulas in Getzler's proof of the Index theorem.

1984-17

October 15, 16: Construction of the heat operator by constructing its kernel invariantly using the filtration idea.

October 18: Formal construction of e^{-tH} at $t = 0$ using the nilpotency of $t, (t^2 = 0)$. Relations between Berligne-Vergne forms and Chern forms.

October 19, 20: Attempt to understand how to construct the Thom form for a complex vector bundle E using $\mathbb{P}(E) = S/S^1$ and $\xi^n + c_1(E) + \dots + c_n(E) = 0$. Review Gaussian-Thom form coming from the super-connection formalism.

October 21: Chern classes of representations.

October 22, 23: The formula $U = \det e^{-d\bar{z} \frac{1}{\bar{w}} dz} e^{-|z|^2}$ for the Thom form. Pfaffian and related formulas.

October 26: Constructing heat operators e^{-tH} where H is a Laplacean type operator. Review of Atiyah's lecture on mixed volumes and isoperimetric inequalities.

October 27, 28: Moment map calculations. Relations between convex bodies, toral varieties and the moment map as defined by Atiyah.

October 29: Strook's remarks on Stratonovich's versus Ito's stochastic differential equations.

October 31, November 2,3: Constructing $e^{t\Delta}$ where $\Delta = \rho^{ij} \partial_i \partial_j +$ lower order terms.

November 4: The algebra of smooth kernels in the tangent groupoid.

November 5: Equivalent descriptions of a singularity.

November 6,7: Thom form and Bott's residue calculations.

November 8: Feynman's formula: $\frac{1}{ab} = \int_0^1 \frac{dt}{((1-t)a+tb)^2}$.

November 9: Singular transgression form.

November 10: Proof that $c_1(\theta(1) \otimes \pi^* E) = 0$ in $H^*(PE)$ using differential forms.

November 11: Physicist's integration process for differential functions on the loop space \mathcal{LM} .

November 12: Constructing heat operators.

November 13: Reflection positivity.

1984-18

November 14: Review of geometric approach to the heat operator.

November 15: Homogeneous distribution on \mathbb{R}^n . Kernels on the tangent groupoid.

November 17: Gaussian measure. Current operators.

November 18: Return to construction of $e^{t\mathbb{D}+\theta D}$. Bismut's separation of parallel transport in $S \otimes E$ using Brownian motion.

November 19: Summary of links between the fermion integral, Bismut's use of Brownian motion in $\text{Lie}(\text{Spin}(n))$ and Vergne's Laplacian in the group direction.

November 20: Roe's thesis.

November 22: Comments on the map $\Omega(\mathcal{L}M)^{S^1} \rightarrow \Omega(M)$.

November 26: Notes on the basic 2-form on $\mathcal{L}M$ coming from the Riemannian structure on M .

November 27-30: Integrating differential functions on $\mathcal{L}M$.

December 1: Fermion integration formulas.

December 2: On the skew form $w(f, g) = \int_0^1 fg' dt$, a generalization and the fermion integral.

December 5,6: Weiner process on the line and related problems.

December 7: Notes on Bismut's construction. Bosonic and fermionic algebras.

December 8-11: Gaussian measure.

December 12: Equivariant cohomology $H_{S^1}(\mathcal{L}BU(1))$.

December 19-26: On $\Lambda(V)$ and why $\det(w)e^{-\bar{J}\frac{1}{w}J}$ is denominator free. Pfaffian algebras and details for a paper with Matthei.

1984-19

Lecture notes on infinite determinants.

1984-20

Notes on lectures by Connes and Hurder.

Contents 1985

1985-1

- February 6,8, 25: Cramer's theorem and the Weak Law of Large Numbers.
- February 28: Conditional expectation. Martingales.
- March 6: On a map from the cyclic cohomology of A to the Lie algebra cohomology of $\mathfrak{gl}_n(A)$.
- March 8: Hölder's inequality. Doob's inequality.
- March 9: Notes for a lecture on invariants of \mathfrak{gl}_n .
- March 10: Differential forms and the Bott map.
- March 11: Periodicity maps using Clifford algebras.

1985-2

- March 13: Periodicity maps using Clifford algebras and character functions for the Grassmannian.
- March 14: Beginning of a proof of homotopy invariance.
- March 15,17: Cyclic cocycles and idempotent operators F - Connes and Quillen approaches.
- March 18, 20: The Atiyah-Singer proof of periodicity based on Kuiper's theorem.
- March 21: Witten's graph construction.
- march 23: If $f : \mathbb{N}_{>0} \rightarrow \mathbb{R}$ satisfies $f(m+n) \geq f(m)f(n)$ then $\lim_{n \rightarrow \infty} f(n)$ exists in $(-\infty, \infty]$.
- March 24, 25, 26: Fatou's lemma and an application using the Fenchel transform. Review of convex analysis.
- April 5,6: Gaussian discrete random processes and martingales.
- April 8,9: Martingale a.e. convergence theorem.
- April 12: Martingales and Markov chains. Integration with respect to a martingale (martigale transform).
- April 14: Positive super-martingales and potentials.
- April 15: Dubin's inequality.
- April 17, 19: Upcrossing inequalities for martingales. Martingale inequalities.
- April 20: Definitions and theorems for martingales.

1985-3

- May 9, 11, 12: Eigenvalues of stochastic matrices and positive matrices. Von Neumann L^2 -ergodic theorem.
- May 14, 16: Links between large deviations, the ergodic property and the Sanov theorem (as described by Strook). Large deviations for Markov chains.
- May 25: Doubly stochastic matrices and map $T \backslash U(n) / T \rightarrow$ doubly stochastic matrices.
- June 2, 6: Classical gas review.
- June 7: Curie-Weiss model for magnetism in relation to large deviation ideas.
- June 8: Strook's large deviation result for finite Markov chains.
- June 9: What is the infinitesimal version of a positive matrix amd the corresponding Frobenius theorem? Continuous analogue of a stochastic matrix.

June 11: Interpretation of effective potential.

June 12, 13, 14, 16: Ruelle's results and relationships between Fried's paper, Markov partition functions and ζ -functions.

June 20, 21, 22, 25: Quantum Hall effect and flat line bundles.

June 24: Quantum Hall effect and the irrational rotation algebra (as described by Connes).

1985-4

June 26 - July 3: Quantum Hall effect and line bundles on \mathbb{R}^n/Γ .

July 4, 6, 7: Kroneker foliation and irrotational rotation algebra \mathcal{A}_θ .

July 8, 9, 10: Loop group representation and Deligne's dilogarithm pairing. Comment on the L^2 Index theorem for $\bar{\partial}$ on a line bundle over \mathbb{C} .

July 15, 16: Araki-Woods construction.

July 21: Graeme Segal's construction of the fundamental representation of $\mathcal{L}(U(n))$ from that of $\mathcal{L}(U(1))$.

1985-5

August 3: Comments on the theory of 1-dimensional Gaussian integrals.

August 5: Problem of the phase space path integral for the simple harmonic oscillator.

August 6: Infinite dimensional Gaussian measure.

August 7: Shale-Strinespring theory. Quadratic elements of Fock space. How $\int x_t \dot{x}_t$ for Brownian motion is not canonically defined in Fock space.

August 9: Links between functional integrals and quantum mechanics.

August 10: Osterwalder- Schrader reconstruction of a quantum field theory from the Schwinger functions.

August 11: Multiple harmonic oscillator and the Gaussian integral on a space of paths in \mathbb{R} .

August 12: Continuous analogue of the moment problem.

August 13: Review of some gas theory.

August 14, 15: Gaussian Markov chains. Osterwalder-Schrader theorem and reflection positivity.

August 16, 19, 20: Path integral representation for quantum theory and the Ising model.

August 21: Atiyah-Singer periodicity theorem. Periodicity maps $\Omega\mathcal{F}_1 \sim \mathcal{F}_0$.

1985-6

August 22: Periodicity $\Omega\mathcal{F}_1 \sim \mathcal{F}_0$ and $\Omega U \sim \mathbb{Z} \times BU$ at the level of Dirac operators.

August 24: Loops of Dirac operators and Kasparov product. Volume of spheres and $\zeta(s)$.

August 26: Can $\log \theta(\frac{1}{t}) = \text{analogue of } h^0(L)$ be interpreted as a dimension?

August 27: Gaussian integrals in infinite dimensions.

August 28: Morita equivalence of \mathcal{A}_θ and $\mathcal{A}_{\theta^{-1}}$. Does $\partial_x + 2\pi x$ define a K -homology class for the Weyl algebra?

August 29, 30: Free fermion gas associated to $H = -i\partial_x$ in $L^2(S^1)$. Obtaining and using partition functions. Does Clifford multiplication extend to Fock space?

August 31, September 1,2: Gaussian states on a Clifford algebra. Araki-Woods boson construction leading to non-type I factors.

1985-7

Quillen's index of 1985-7 and 1985-8 files.

September 23: On a simple harmonic oscillator with time dependent forcing.

September 25: On flat bundles over the complex torus $\mathbb{R}/\beta\mathbb{Z} \times \mathbb{R}/L\mathbb{Z}$.

September 26: A family of Fock spaces associated to a family of flat line bundles over S^1 , with a construction of charge and energy.

September 27, 28: Review of $\det(\partial_t + \frac{1}{i}\partial_x + F)$, $\text{tr}(U_f(\beta, 0))$ computations.

September 30: Bosonic picture with $\text{tr}(U_f(\beta, 0))$ as a bosonic path integral.

October 2: Review vertex operators. Deligne/ Bloch construction of a regular map $K_2(X) \rightarrow h^1(X, \mathbb{C})$ where X is a Riemann surface.

October 3: Weyl algebra attached to a loop on a Riemann surface.

October 4: Thermal states for the simple oscillator.

October 11: Ideas and analogies.

October 24, 25: Phase space path integral for the simple harmonic oscillator and complex Gaussian integrals.

October 30: Discussion of complex quadratic space, a real form and Gaussian integral.

November 13: Ideas for a continuous analogue of the Weil theory over \mathbb{F}_q and possible applications to the Riemann zeta function. Continuous analogue of $\frac{1}{\det(1 - zA)}$.

October 14: Bernoulli polynomials and Γ -function formulas.

1985-8

November 29: Non-commutative residue.

December 6,7,8: Wodzicki's determinant for elliptic pseudo-differential operators.

December 10: Seeley's work on $\zeta_A(s)$ and Wodzicki's work.

December 11, 14: Symbols and kernels for pseudo-differential operators. Constant coefficient operators on a torus.

December 16, 17: Why does Seeley theory give information on values of ζ_A at negative integers?

December 18, 19: Gaussian measures and Malliavin's infinitely differentiable functions on Wiener space.

December 20: Malliavin Sobolev spaces.

December 24, 25, 26: On Lewis's talk (in Dublin) on quantum stochastic processes. Transmission line and a tuned circuit, classical solution S -matrix, energy, symplectic structures.

December 27: Replace transition line by a loop group representation setup.

December 28: Standard model emerging from loop group representation.

December 29: Non-commutative Gaussian processes.

Contents 1986

1986-1

- January 4: \mathbb{C} -valued Gaussian processes.
- January 5: Gaussian integrals with complex exponents.
- January 6: Gaussian integrals in infinite dimensions with non-real exponents.
- January 8, 9: A Gaussian measure on finite dimensional space V is a state in the Weyl algebra $W(V \oplus V^*)$.
- January 10: Equivalences associated to a symplectic vector space.
- January 13, 14: Summary of work so far this month.
- January 15: KMS condition and Gaussian states on a Weyl algebra. Fermion case.
- January 16: Further discussion of Gaussian measures on infinite dimensional spaces.
- January 17: A distribution theory viewpoint towards Gaussian measures on infinite dimensional spaces.
- January 18: Families of Gaussian probabilities $\mu_h = e^{-\frac{1}{2h^2}|x|^2} dx / \text{norm}$ and Fock spaces. Solutions to the heat equation $\partial_t f = (\partial_z^2 + z\partial_z)f$.
- January 20: Rescaling.

1986-2

- Quillen's own index January 30, 1986 - March 11, 1986.
- January 31: February 1,3: Quantum mechanics of the forced harmonic oscillator.
- February 5: Gaussian integrals with complex exponents.
- February 7: Discrete serie of $\widetilde{SL(2, \mathbb{R})}$. Direct construction of unitary representation of $\widetilde{SU(1, 1)}$.
- February 10,11: General discussion of Gaussian integrals with complex exponents.
- February 13: Gaussian integrals such as $\int e^{-\int \bar{\psi}(\partial_t + w)dt} (?) \mathcal{D}\bar{\psi} \mathcal{D}\psi$.
- February 14: Lagrangian subspaces of a symplectic vector space. Cyclic cohomology of a Weyl algebra.
- February 15: Cyclic cohomology of a Weyl algebra.
- March 1: Connes construction of cyclic cocycles attached to a Dirac operator using a transgression operator.
- March 2,3: Transgression of the closed 1-form $\text{tr} \left(\frac{1}{1-L^2} dL \right)$ where $L = \partial_x + \alpha$ over S^1 .
- March 4: Smoothing a singularity by a Connes trick. Construction of transgression using a smoothed Heaviside function.
- March 10: Calculation of $\phi(a) = \text{tr}(T^{-1}[a, T])$.
- March 8: Discussion of transgression problem.
- March 11: Construction of cyclic cocycles $\text{tr}(F[F, a][F, b]) = 2\text{tr}(a[F, b])$, where F is an involution.

1986-3

- Quillen's own index March 12 - April 20
- March 12: Atiyah-Singer proof of Bott periodicity using spaces of Fredholm operators.

March 14: The map $U_{\text{res}}(H) \rightarrow U(B^+/K^+)$ is a homotopy equivalence.

March 15: Spectral theorem for subspaces $W \subset V^+ \oplus V^-$. On the Cayley transform from self-adjoints commuting with $\epsilon = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ to unitaries carried into their inverses by ϵ .

March 16: The restricted unitary group is homotopy equivalent to the space of Fredholm operators.

March 17, 18, 19: Gaussian or quasi-free states on the Weyl and Clifford algebras. A simple proof that the restricted unitary group is homotopy equivalent to the unitary group of the Calkin algebra. Dilating A , where $-1 \leq A \leq 1$, to an involution.

March 20: Forming a graded involution from an ungraded self-adjoint operator forming an ungraded involution from a graded self-adjoint operator. Massive ‘Dirac’ operator. Square root of the Cayley transform.

March 21: Maps $\mathcal{F}_0 \xleftarrow{\text{Toeplitz}} U_{\text{res}} \rightarrow \mathcal{G}_{\text{res}}$.

March 22: Atiyah-Singer periodicity proof using Cayley maps.

March 23: Identifying involutions with a set of unitary maps. Higher K -groups identified as lower K -groups of certain categories rather than as homotopy groups.

March 24: Canonical involution mod \mathcal{K} in $L^2(M, S \otimes E)$. Graded versions of U_{res} .

March 25: Formula for the pull back of $\text{tr}(F(dF)^k)$ on $\mathcal{J}(H)$ to $U(V)$.

March 26: Index in K -theory for a family of Dirac operators parametrized by \mathcal{A}/\mathcal{G} consistent with the canonical map $\mathcal{G} \rightarrow U_{\text{res}}$.

1986-4

March 27: Index of a family of Dirac operators over a circle and Kuiper’s Theorem.

March 28: Milnor classifying space. Kuiper trivialization and associated constructions.

March 29 - April 2: Index of a family of Dirac operators on S^1 parametrized by $U(n)$, as an element in $K^1(U(n))$.

April 4: Connes connections $\nabla : \mathcal{E} \rightarrow \mathcal{E} \otimes_{\mathcal{A}} \hat{\Omega}^1$.

April 6-13: Linking even forms on U_{res} and odd forms on \mathcal{F}_1 .

April 16-19: Promising approach to transgression problem. Restricted Grassmannians.

April 20: Graeme Segal’s ideas for constructing an extension of $U_{\text{res},(0)}$ by $U(\mathcal{K})$. Natural frame bundle over the Grassmannian $\mathcal{J}_{\text{res},(0)}$ for $U(\mathcal{K}^+)$.

1986-5

Quillen’s own index April 21-June 24, 1986.

April 21, 22: Letter to Mathai on excision in K -theory. Index over $B\mathcal{G}'$ classifying pairs (E_0, E_1) of Hilbert bundles with an index zero isomorphism modulo compact operators. Explicit models for BU_{res} and index maps.

April 23: Index map for pairs of projections congruent modulo compact operators.

April 24: Transgression.

April 25: More work on U_{res} and associated classifying spaces.

April 26, 27: Constructing a 1-form on a model for $B\mathcal{G}$ consisting of a projection on $H^+ \oplus H^-$ commuting with the grading ϵ .

April 29: On a map $B\mathcal{G}' \rightarrow \mathcal{F}_0$.

April 30: Alternative to the Bott map $\text{Gr}(V) \rightarrow \Omega(U(V), 1, -1)$ and formulas for characteristic forms.

May 2: Laurent polynomial loops. Ideas for an alternative proof of periodicity.

May 5: Real periodicity.

May 7: Brief discussion of a model for $B\mathcal{G}'$ and an old problem of linking super-connections with Grassmannian graph methods.

May 8: The 1-form on $B\mathcal{G}$.

May 9: Berry's (Bristol) talk on quantum chaos of the Riemann zeta function. Another model of $\Omega(\text{Grass})$.

May 11: Transgressing cyclic cocycles on the restricted unitary group to differential forms on a model of its classifying space.

May 14: Maps from connections on $C^\infty(S^1, \mathbb{C}^n)$ to \mathcal{I}_{res} .

May 16: Connections on the canonical bundle over $U_n \times S^1$. Linking odd character forms for U_n with even character forms on $\mathcal{G} = \mathcal{L}(U_n)$.

1986-6

May 18: Lifting a character form γ_{2k+1} on U_n and writing it as $\gamma_{2k+1} = d\mu_{2k}$ where μ_{2k} is \mathcal{G} invariant.

May 19: Loops on the Grassmannian and the loop group \mathcal{G}^σ .

May 22, 23, 26: Using infinite repetition to construct $B\mathcal{G} \rightarrow \mathcal{I}(\mathcal{Q})$, where $\mathcal{G} = U_{\text{res}}$.

May 29: More on the problem: given a Fredholm module (\mathcal{A}, H, F) to construct differential forms on the space of projectors over \mathcal{A} .

June 1: Left invariant forms on $\mathcal{G} = \text{Aut}(E)$ and embedding $E \hookrightarrow E'$.

June 3,7: Understanding the Bott map and the S -operator.

June 8: Linking the S -operator with periodicity.

June 11: Existence of a fibration $\mathbb{Z} \times BU \rightarrow * \rightarrow U$. Quasi-fibration $\mathcal{I}_{\text{res}} \rightarrow \mathcal{F}_{1,\eta} \rightarrow U(\mathcal{K})$.

June 13: Review of odd forms on \mathcal{G} . Families of Dirac operators on S^1 .

June 14: Review of what is known about producing character forms on \mathcal{G} , including Bismut forms.

June 17, 18: Proof of periodicity based on quasi-fibration ideas.

June 19: Review of an unsolved problem of constructing forms on a Grassmannian.

June 22: A third way of constructing forms, closely related to Bismut's construction.

June 23: Exactness of forms $\pi_*(\text{tr}(e^{\Delta_X + \nabla}))$.

June 24: Odd forms on $B\mathcal{G}$, where $\mathcal{G} = \mathcal{L}(U_n)$, defined by holonomy, are special cases of those formed by integrating over S^1 the character forms of a connection on $B\mathcal{G} \times S^1$.

1986-7

Quillen's own index of July 11-Sept 12, 1986.

July 11, 18: Linking super-connections with Grassmannian graph ideas. Curvature of graph subbundles of $\text{Gr}(E^0 \oplus E^1)$.

July 22: More on the Grassmannian graph.

July 23: Review of Bott-Chern forms.

July 24: Connes - Moscovici transgression.

July 25, 26: On the limit graph: for $V_0, V_1 \cong \mathbb{C}^n$, $\phi_t = \begin{pmatrix} 1 & 0 \\ 0 & t \end{pmatrix}$ on $V_0 \oplus V_1$, $Z_t = \text{graph}(\phi_t)$, $\lim Z_t = Z_\infty \subset \text{Gr}(V_0) \times \text{Gr}(V_1)$.

July 29, 30: Computing cocycles in Gr .

July 31, August 2: On correspondance $\text{Gr}_n \leftarrow V \rightarrow \mathbb{P}V^0 \times \tilde{\mathbb{P}}V^1$.

August 3: Calculations related to a correspondance
$$\begin{array}{ccc} \{(K, I, \Gamma, W)\} & \rightarrow & \text{GL}_n(V) \\ & \downarrow & \\ & \text{GL}_n(V) & \end{array}.$$

August 4: Further calculations on $\text{Gr}_n(V^0 \times V^1)$. Decomposition of cohomology of the Grassmannian and correspondances effecting it.

August 5: Jacobi triple identity and decomposition. Conversation with Graeme Segal on proofs of $\frac{1}{\prod_{i>0}(1-q^i)} = \sum_{s>0} \frac{q^{s^2}}{(\prod_{i=1}^s(1-q^i))^2}$.

August 6: Composition of correspondances. Where does class c_k come from?

1986-8

August 16: Natural correspondances between $H^*(\mathbb{Z} \times BU)$ and the fermion Fock space of $L^6(S^1)$, and the relationship between the Jacobi formula and a previous decomposition of $H^*(BU)$.

August 17: Integration over the fibre for $D_{1\dots d}(V) \rightarrow \text{Gr}_d(V)$.

August 18: Canonical isomorphism $\Lambda^k(H^*(\mathbb{P}V)) \simeq H^*(\text{Gr}_k(V))$.

August 19: Review of Berry's talk on the Grand Unitary Ensemble.

August 20, 21, 22: Grassmannian graph ungraded case. Morse decomposition. Twisted case occurs as a desingularisation for the untwisted case. The character lies in the $s-1$ in the twisted case.

August 25: Chern-Simons difference form. Investigating $\lim \phi^*(\gamma_k)$ where γ_k is the k th character form.

September 2: Super-connection forms associated to $V_0 \oplus V_1$.

September 3: Super-connection forms are globally defined on the Grassmannian bundle.

September 5: $\lim_{\lambda \rightarrow 0} \text{tr} \left(\frac{\sqrt{\lambda}}{\lambda - X^2} dX \right)^{2k}$ near skew-hermitian X_0 of corank 1.

September 7,8: Limit of odd character forms for $U(N)$.

August 8: Calculations and proofs related to the limit of odd character forms.

September 10,11: Restriction property for super-connection forms.

September 12: Review of material on Γ -function. Links between asymptotic expansion of $\text{tr}(e^{-tA})$ as $t \rightarrow 0$, residues of $\zeta_A(s) = \text{tr}(A^{-s})$ and asymptotics of $\text{tr} \left(\frac{1}{\lambda - A} \right)$ as $\lambda \rightarrow \infty$. Euler-Maclaurin formula.

1986-9

September 15: Connes-Moscovici form: $w_k = \text{tr}_S \left(\frac{1}{1-X^2} dX \right)^{2k}$ where $\begin{pmatrix} 0 & -T^* \\ T & 0 \end{pmatrix}$.

September 17: Review of April's work and understanding a letter from Connes.

September 18 - 23: Transgression problem and super-connections.

September 24: Family of operators parametrized by $\Sigma(\mathcal{G})$ where $\mathcal{G} = \text{Aut}(E_0)$. Graeme Segal's ideas on using spectral flow.

September 26, 27: Using adiabatic approximation and loop group $(\mathcal{L}U_{2n})^\sigma$.

September 28: Canonical Hilbert bundle over $\Sigma(\mathcal{G})$ and KK -theory.

September 29: Relating family over $\Sigma(\mathcal{G})$ to the map $\mathcal{G} \rightarrow \mathcal{I}_{\text{res}}$ via periodicity in K -theory.

September 30: Adiabatic approximation.

1986-10

October 4: KK -theory and the Hilbert bundle over $\Sigma\mathcal{G}$.

October 5: KK description of an element of $K^1((0, 1))$ and the operator $\partial_t + A_t$ on $L^2((0, 1), H_0)$.

October 7: Ihara's talk on cyclotomic units.

October 11, 12: Is the model of the loop group for Grassmannians useful for the transgression problem?

October 16: Review of the Index theorem over S^1 from Shanahan's notes.

October 17, 19: Return to family of Dirac operators over $\text{Gr}(E)$.

October 20: Adiabatic expansion. Return to Dirac operators over S^1 .

October 21: The index of a family of Dirac operators over M gives a class in $K(B\mathcal{G})$, where \mathcal{G} is the group of gauge transformations, transgress to give a K -class on \mathcal{G} . Relationship to super-connections on $B\mathcal{G}$.

October 22: Symmetric representation $\text{Gr}(V^0) \times \text{Gr}(V^1) \rightarrow \text{Gr}(V^0 \oplus V^1)$.

October 23, 24: Non-adiabatic limit. Calculating $\frac{1}{t}\partial_t + (1 - \rho(t))\not{D} + \rho(t)g^{-1}\not{D}g$ where ρ is a smooth approximation to the Heaviside function.

October 26: Return to loop space model of the Grassmannian.

October 27, 28, 29: Bound states for the operator $\begin{pmatrix} 0 & -\partial_t + F_t \\ \partial_t + F_t & 0 \end{pmatrix}$ where the F_t are certain involutions.

October 31: Evaluating the loop group model for $\Omega(\text{Gr}(\mathbb{C}^{2n}); \epsilon, -\epsilon)$.

1986-11

Quillen's own index for November 1, 1986 - January 28, 1987.

November 1: Remark on another approach to Connes S -operator.

November 5: For unitary $g \in U_2$, producing a canonical deformation u_t of $\begin{pmatrix} g & 0 \\ o & g^{-1} \end{pmatrix}$ such that u_t , for $t \neq 0$, has no eigenvalue equal to -1, and possible constructions of the periodicity map.

November 6: For graded (E, D) associate forms to each $\Gamma \subset E$.

November 7: Remark on unitary group flow.

November 9: On $\log(\lambda - A)$ where A has its spectrum in $\mathbb{R}_{\leq 0}$.

November 10: Isomorphism of superalgebras $A \hat{\otimes} B \simeq A \otimes B$.

November 11, 12: Super-connection $\epsilon D + X$

November 13: Is $\log(\lambda - X^2 - dX\sigma) - \log(\lambda - X^2)$ defined on the unitary group?

November 18: Account of conversation with Friedan on conformal field theory.

1986-12

November 20: On the form $cw_{2k+1}^t = (4t)^{2k+1} \text{tr} \left(\frac{1}{t(g+1)+g-1} dg_{\frac{1}{t(g+1)-(g-1)}} \right)^{2k+1}$ where $g \in -U_p(H) = \{g \in U(H) : g \equiv -1 \pmod{L^p(H)}\}$. Narashiman-Ramanan theorem.

November 21,25: More on the Narashiman-Ramanan theorem. Proof for a non-compact space. Character forms for $-U_p(H)$.

November 26: On the differential form $R_\lambda = \frac{1}{\lambda - X^2 - dX\sigma}$.

November 28: Normalizing character forms.

December 1: Proof that smooth $\phi : S^1 \rightarrow \mathbb{C}$ gives smooth $g \mapsto \phi(g)$ from U_p to $\phi(1) + L^p$.

December 3: Discussion of Connes-Moscovici form from Quillen's viewpoint.

December 5,9: $U^p(H)$ and $\text{Gr}^p(H, \epsilon)$ are Banach manifolds.

December 13: Polynomial growth in $L^{p'}$

January 4, 1987: On a simple proof of the Index theorem by imbedding methods. Resolvent $\frac{1}{\lambda - L^2}$ for $L = h\gamma^\mu \partial_\mu + \sigma X$.

January 6, 7: To extend the Dirac operator $L = \not{\partial} + X\sigma$ to the Cayley transform of X . Von Neumann's construction.

January 20: Transmission lines and strings.

January 21, 23: Summary of some symplectic geometry and the manifold of Lagrangian subspaces.

January 24: Discussion of $L = \begin{pmatrix} 0 & \partial_x - f(x) \\ \partial_x + f(x) & 0 \end{pmatrix}$ where $f : S^1 \rightarrow \mathbb{R}$ is smooth.

January 25: Transmission lines and Krein strings.

January 28: Discussion of $\gamma^\mu \partial_\mu + \sigma X$ where X is allowed to become infinite.

Contents 1987

1987-1

Quillen's own index for January 11 - March 2, 1987,

January 29: K -theory cup product using Clifford addition and multiplication.

January 31, February 1: The Gysin homomorphism in K -theory - integration along a fibre. Properties of $L = \begin{pmatrix} 0 & \partial_x - f \\ \partial_x + f & 0 \end{pmatrix}$.

February 3, 5: Quillen's commentary and related thoughts on the paper by Goddard and Olive: Kac- Moody and Virago Algebras in relation to Quantum Mechanics.

February 7: Transmission lines and strings. Singular Dirac operators and resolvent.

February 8, 9: Reformulation of transmission line equations in terms of scattering data.

February 10: Using Clifford algebras to understand the normal ordering process.

February 11: More on normal ordering and Wick's theorem. Action of Virsoro algebra.

February 12: Vertex operators.

1987-2

February 13: Clifford algebra of $L^2(S^1)$ with the quadratic form $\frac{1}{2} \int f^2 \frac{dx}{2\pi} = \frac{1}{2} \sum f_k f_{-k}$ where $f = \sum f_k e^{ikx}$, and fermion fields. Virasoro algebras. Commutation relations.

February 14: Operator product expansion for stress energy operator. Virasoro algebra calculation.

February 15, 16, 17: List of various approaches towards the existence of the resolvent $\frac{1}{\lambda - z}$ for $z = h\gamma^\mu \partial_\mu + \sigma X$ where $X = \frac{g-1}{g+1}$. Formulas for Virasoro algebras and $\frac{1}{2}$ -densities.

February 18: Discussion of quantum field theory on Riemann surfaces.

February 19: More Virasoro calculations.

February 22: Ising model.

February 23: Clifford algebra calculations.

February 25, 26: Dirac operator over a circle with super-connection coefficients. Doubling.

February 27: Notes for Roe on $h\gamma^\mu \partial_\mu + \sigma X$ followed by a discussion of the same operator.

February 28: Skew-adjoint operators and subspaces. Approaches to constructing U_g where $g : M \rightarrow U(V)$ and U_g acts on $L^2(M, S \oplus V)$.

March 1: Finish example $Q_X = h\gamma^1 \partial_x + \gamma^2(-ia)$ where $a \in \mathbb{R}$.

March 2: More notes for Roe on Von Neumann's approach to unbounded skew-adjoint operators and $g \mapsto U_g$ as for Feb. 28.

1987-3

Quillen's own index for March 4 - April 21.

March 4: Review of various facts about symplectic structures.

March 10: Conformal fields theory on Riemann surfaces.

March 11: Real fermion fields on the torus.

March 12: The cohomology of a holomorphic vector bundle using meromorphic and adelic ideas.

March 13: Witten's QFT on a Riemann surface.

March 14: Complex Fermi field. Imaginary time evolution.

March 15, 17: On a vector space of operators associated to rational functions of z .

March 18: Spinors over \mathbb{P}^1 . Fock spaces attached to a Riemann surface.

March 19, 20: Fock space attached to a point.

March 21: On a Riemann surface with a single boundary component.

March 22 : Raymond case. Irreducible representations of Clifford algebras and Fock space spinor fields.

March 24: Review of facts about Clifford algebras.

March 26: Structure of arguments used for the Ising model.

March 27: The Ising model.

March 28, 29: Fermionization in the Ising model.

1987-4

March 30: Discrete analogues of spinors over the circle.

April 8: Getzler and Toeplitz operators.

April 9: Pairing the Dirac operator on the circle with a loop $g : S^1 \rightarrow U(V)$ using Kasparov KK -theory cup product.

April 10, 11: Yesterday's pairing leads to a Dirac operator on the torus.

April 12: Kasparov Hilbert C^* -module point of view applied to pairing a Dirac operator on the circle and a loop $g : S^1 \rightarrow U(V)$.

April 13 - 17: Pseudo-differential operators on S^1 .

April 18: Notes on the map $S^1 \times S^1 \rightarrow S^2$ related to cup product in K -theory, and non-smoothness.

April 19: Smooth version of the cup product for $U(V) \times U(W) \rightarrow Gr(\mathbb{C}^2 \otimes V \otimes W)$. Notes for J. Roe on a Kasparov type construction for pairing an odd K -cohomology class of S^1 with the fundamental K -homology class.

April 21: Counter-example encountered in writing the notes for J. Roe.

1987-5

April 27: Torus example. Limiting behaviour of the graph of $\bar{\partial}$.

April 29: Graph of $\partial_x + bx$ for large b . Proofs of Witten's result.

April 30, May 1: Review of problem of coupling a loop $g : S^1 \rightarrow U(V)$ to Dirac ∂_x on S^1 .

May 2: Approximating $\partial_x \pm x$ on \mathbb{R} by an operator on $(-\delta, \delta)$.

May 3: Analysis of the 'blip' operator given by a loop $g : S^1 \rightarrow (U(1))$ of winding number 1, concentrated near 1.

May 4, 5: Construction of a certain pseudo-differential operator of index 1 on S^1 .

May 6: Coupling the Dirac operator on S^1 to a unitary loop. Loring's projector.

May 7: Papers on bosonization on a Riemann surface by Bost-Nelson, Alvarez, Gomez and Vafa.

May 8: Parametrix methods applied to the coupling problem.

May 9: Some analysis of elliptic operators.

May 10: Invertibility of $s - \not{D}$ where $\not{D} = \begin{pmatrix} 0 & \partial_x - q \\ \partial_x + q & 0 \end{pmatrix}$ for $|\operatorname{Re}(s)|$ large.

May 11: More on pairing the Dirac on a circle with a loop.

May 12: On Weil's Acta paper about an adelic version of the θ -function and a proof of quadratic reciprocity associated to $K_2(\mathbb{Q})$.

1987-6

May 13: On Weil's Acta paper.

May 14: Analysis of a real scalar field ϕ on S^1 with Hamiltonian $\underbrace{\int \frac{1}{2} \phi^2 dx}_{P.E.} + \underbrace{\int \frac{1}{2} \phi (-\partial_x^2 + m^2) \phi dx}_{K.E.}$.

May 26: Bosonization and Graeme Segal's ideas on QFT for holomorphic functions.

May 28: Log-harmonic functions.

May 29 - June 3: Work on bosonization.

June 5, 6: The annihilator of $\Gamma(X, \theta)$ is $\int \Gamma(X, \Omega^1)$ (Kronheimer) and related results. A line in the Fock space of $C^\infty(S^1)/\mathbb{C}$ associated to a Riemann surface with boundary circle S .

June 7: Symplectic vector spaces and self-dual lattices.

June 8, 9, 10: On a real symplectic vector space and its Heisenberg representation.

June 15, 26: On the cohomology of a Riemann surface with g handles and r boundary circles. More on QFT.

June 17: Further discussion of a real symplectic vector space and its Heisenberg representation.

1987-8

June 18, 19: Functions and 1-forms on a Riemann surface with boundary.

June 20, 21: Cohomology of the double of a Riemann surface with boundary. Riemann's conditions for a closed Riemann surface.

June 22 -25: Calculations for a complex vector space with a Hermitian form or a symplectic form.

June 26 - July 2: Quasi-free states for a real symplectic vector space with a skew form $S(x, y)$ and Weyl algebra with $[x, y] = iS(x, y)$.

July 4: On θ -functions.

July 5: The link between Riemann type θ -functions and intrinsic θ -functions.

July 6: Note on a function satisfying a quasi-periodicity property.

July 7, 8, 9: The category of real finite dimensional vector spaces and complex symplectic morphisms.

July 10: An intrinsic θ -function on a complex vector space with a Hermitian inner product and a lattice.

July 11: The Heisenberg representation of an abelian extension of a self-dual lattice. Review of some group cohomology. More on relating the intrinsic θ -function to a Riemann θ -function.

July 13, 14: An account of θ -functions.

1987-9

July 15, 20: Coupling super-connections with Dirac operators.

July 21, 22, 23: On the equation $(p^{\frac{1}{2}} F p^{\frac{1}{2}} + iq)\phi = f$ with $p = g^{-\frac{1}{2}} \left(\frac{q+1}{2} \right)$, $q = g^{-\frac{1}{2}} \left(\frac{q-1}{2} \right)$.

July 26: The operator $g^{\frac{1}{2}}(pF + iq) = \left(\frac{q+1}{2}\right)F + \left(\frac{q-1}{2}\right) = gP_+ - P_-$ is an elliptic pseudo-differential operator of order 0.

July 31, August 1, 2: Differential operator case, looking at the subspace $\text{Im} \begin{pmatrix} r \\ p\partial_x + q \end{pmatrix}$ and calculations with the Green's function.

August 2, 3: On the ordinary differential equation $u - pu' = f$.

August 5: Transmission lines.

August 6: Putting in Planck's constant.

August 7: On the Dirac operator in the form $\mathcal{D} = \begin{pmatrix} 0 & h\left(\frac{p}{r}\right)^{\frac{1}{2}}\partial_x\left(\frac{p}{q}\right)^{\frac{1}{2}} - \frac{q}{r} \\ h\left(\frac{p}{r}\right)^{\frac{1}{2}}\partial_x\left(\frac{p}{r}\right)^{\frac{1}{2}} + \frac{q}{r} & 0 \end{pmatrix}$.

August 8: Parametrix methods.

August 13: Note on defining a subspace of $L^2(\mathbb{R})^2$ by $\text{Im} \begin{pmatrix} r \\ L \end{pmatrix}$ where $L = p\partial_x + q$.

August 14: On minimal and maximal closures of densely defined operator.

August 15: On the equation $D_x P(x, y) = \delta(x, y) - K(x, y)$ where D is a differential operator, P a distribution smooth off the diagonal and K a smooth function.

August 16: On Schur's estimate.

1987-10

August 23: Friedrich's mollifier methods.

August 25 - September 5: Coupling ∂_x with an even K -class.

September 6, 7: Review of problem of given a unitary matrix function g to find a connection d_B such that if ρ_1 and ρ are suitable functions on S^1 vanishing at -1 then $\rho_1(g)^{-1}(d + B)\rho(g)$ is smooth.

September 8: Alternative approaches to constructing a pairing between ∂_x and g .

September 9, 10: Cup product map $\mathbb{R} \cup \{\infty\} \times \mathbb{R} \cup \{\infty\} \rightarrow \mathbb{C} \cup \{\infty\}$ given by $(x, y) \mapsto x + iy$. Smoothing the singularity.

September 12: Non-abelian case of associating to a loop an index -1 pseudo-differential operator of the form $r(h\partial_x + f)^{-1}$.

September 14 - 17: Arranging for $r \sum_{i=0}^{n-1} f_0^{n-1-i}(\partial_x + f_1)(f_0^i)$ to be smooth.

September 18: From a 2×2 hermitian matrix A , construct a 1-form rf such that $rdA + rfA - Arf$ is of the form $Ts(A)$ with $s(x)$ vanishing to large order at $x = 0$.

September 19: Application of the 2×2 matrix calculation.

September 26, 27: Calculation for when g has a constant number of eigen values equal to -1.

September 28, 29: Review of progress.

September 30: Calculations for $X = \begin{pmatrix} X' & 0 \\ 0 & X'' \end{pmatrix}$ where X'' is invertible and $\mathcal{D} + \sigma X =$

$\partial \begin{pmatrix} \mathcal{D}' + \sigma X' & i^* \mathcal{D} j \\ j^* \mathcal{D} i & \mathcal{D}'' + \sigma X'' \end{pmatrix}$ forming an inverse.

October 1: Review of known methods for coupling ∂_x to an arbitry loop g .

October 2: On the pseudo-differential operator with complete symbol equal to g for $\xi > 0$ and 1 for $\xi < 0$ where $g : S^1 \rightarrow U(V)$ and similarity with the Bott map $S^1 \times U(V) \rightarrow \text{Gr}(\mathbb{C} \otimes V)$.

October 3: Interpreting maps $R \cup \{\infty\} \times (\mathbb{R}/\mathbb{Z}) \rightarrow \text{Gr}(\mathbb{C}^2 \otimes V)$ as points in the restricted Grassmannian.

October 5: Connes groupoid theory and turning a map $T^*(S^1) \rightarrow \text{Gr}(\mathbb{C}^2 \otimes V)$ into an operator.

October 6: Quantizing maps $T^*(S^1) \rightarrow \mathbb{P}^1$ which represent the canonical K -class.

October 7: Deformation of the algebra of functions on $T^*(S^1)$.

October 8: On using K -theory exact sequences. Understanding the map $K_1(A/I) \rightarrow K_0(I)$ in algebraic K -theory.

October 9: Toeplitz operators.

October 10: More on $K_1(A/I) \rightarrow K_0(I)$.

October 11: The index of a projector over the algebra of the deformation.

October 13: Review of the algebras $\mathcal{A}_h = C^\infty(S^1) \otimes \mathcal{S}(\mathbb{R})$ with twisted product $e^{-ix}f(p)e^{ix} = f(p+h)$, and $\mathcal{A} = C^\infty(S^1) \otimes \mathcal{S}(\mathbb{R} \times [0, 1])$.

October 15: Ideas from yesterday's lecture on vector bundles over X/Y .

October 16: More calculations in the twisted algebra including as calculation of the index of an involution.

October 17: Correction of an error on October 11.

October 19: More on an involution $F(h, x, p) = \sum_{n=0}^{\infty} h^n F_n(x, p)$.

October 20: More on \mathcal{A}_h and \mathcal{A} .

October 22: On differential graded algebras. Representation of \mathcal{A} .

October 24: On the index of a pair of idempotents whose difference is compact.

October 25: Multiplier algebra.

October 27: On $\mathcal{A} = q\mathbb{C} = \text{Ker}\{\mathbb{C}e * \mathbb{C}e' \rightarrow \mathbb{C}e\}$ and $\mathcal{A} = \text{Ker}\{\mathbb{C}[\mathbb{Z}/2 * \mathbb{Z}/2] \rightarrow \mathbb{C}(\mathbb{Z}/2)\}$. Trace on \mathcal{A} in $K_0(\mathcal{A})$.

October 28: Fredholm modules.

October 29: More on the algebra of $f(h, x, p)$.

October 30, 31: Formal problem: to compute the index associated to a trace on $\hat{\mathcal{A}}$ and an involution on \mathcal{A}_0 where $\mathcal{A}_0 =$ Schwartz functions on $T^*(M)$ and $\hat{\mathcal{A}} = \mathbb{C}[[h]]$, deformed for the tangent groupoid,.

November 1: Generalizing the $n = 1$ case where the character form is $\text{tr}(F_0(dF_0)2)$ to higher n . Non-commutative differential calculus. $K_0(A) \xrightarrow{\sim} K_0(A/I)$ if I is nilpotent. Calculating K_0 for $\mathbb{C}[\mathbb{Z}/2 * \mathbb{X}/2]$ and the C^* algebra $\mathbb{C}(\mathbb{T}) \times (\mathbb{Z}/2)$.

November 2: Infinitesimal ways of locating elements of $K_0(\mathcal{A})$. KK -theory.

1987-12

November 3, 5: To construct a mechanism for saying that a trace on \tilde{A} (A extended by a nilpotent ideal) becomes equivalent to a cyclic cocycle on A .

November 6, 7: Square zero extensions. Non-commutative Ω^1 . $HC_1(A)$.

November 10: Cyclic homology for a vector space with multiplication.

November 11: Non-unital algebras over a field of characteristic 0.

November 12: Super-algebra context to resolve two approaches to cyclic homology.

November 13: Studying extensions.

November 14, 15: To define a canonical map $HC_2(A) \rightarrow Q/[Q, Q]$ associated to a square zero extension of A : $0 \rightarrow M \rightarrow Q \rightarrow A \rightarrow 0$.

November 16: A process for converting an extension into a Fredholm module.

November 17: On why noncommutative differential forms enter into cyclic homology. Review of relevant K -theory. Cyclic homology of the super-algebra C_1 .

November 19, 20: More on cyclic homology, extensions and traces. Deformation theory of $HC_2(A)$.

1987-13

November 21: On a semi-direct product $Q = A \oplus M$ where $A = T(V)$ is free unital and M is a bimodule over A .

November 22: On the cyclic homology of $A \oplus M$ of a super-algebra A and an A super-bimodule M with $M^2 = 0$.

November 23: There is no canonical map $HC_3(A) \rightarrow HC_1(Q)$ where Q is a square zero extension of A .

November 24: An A -bimodule over $T_A(M)$.

November 25: Goodwillie's proof that if $Q = A \oplus M$ then $\text{Connes}(Q) = \text{Connes}(A) \oplus_{k \geq 1} (\tilde{M} \otimes_A)^k / (1-t)[k-1]$.

November 26: The spectral sequence in cyclic homology for the extension $I \rightarrow P \rightarrow A$.

November 27: Calculation of $d' : E_{k+1,k} \rightarrow E_{k,k}$.

November 30: Exact sequences associated with $P = T(V)$, $P/I = A$.

December 1, 3: Calculations with $T_P(I)$.

December 4: On a Riemann surface with g handles and r boundary circles. Harmonic functions and forms.

December 5: More calculations with $A = P/I$, P free. Product structure on $H^*(A, A)$.

1987-14

Quillens own index for December 6, 1987 - March 11, 1988.

December 6: Normalized acyclic Hochschild complexes.

December 7: Applications of Connes Complex functor.

December 8: Foundations of cyclic theory and an approach different from Connes. Comment on Waldhausen's algebraic K -theory. Hochschild and cyclic homology of free group algebras.

December 9: On the cyclic homology of the algebra A of Schwartz functions on $T^*(M)$ and the algebra P of formal power series with coefficients in A and with a twisted multiplication. Gauss-Manin connection.

December 10: The Gauss-Manin connection.

December 12: Comments on progress in developing cyclic homology.

December 14: The reduced cyclic homology of $a = \mathbb{C}[G]$ where G is a free group.

December 15, 16: Review and a short proof of Goodwillie's theorem.

December 18: More on the Gauss-Manin connection.

December 23: On understanding the map $(L \overset{L}{\otimes}_P)_\sigma^k[1] \rightarrow (I \overset{L}{\otimes}_P)_\sigma^{k-1}$.

December 24, 25: Relative cyclic homology of A relative to P .

December 27: Cyclic homology in terms of cyclic tensor products.

Contents 1988

1988-1

January 16, 17: Goodwillie, Feigin, Tzygan result on the cyclic complex of a semi-direct product $R \oplus M$.

January 22: On the bi-functor $(K, L) \mapsto K \oplus_R L$.

January 23, 24: Calculations with $A = R/I$ where $R = T(V)$. Calculations with $R \oplus \Omega_R^1$, $R = T(V)$. Maps of complexes of R -bundles $\mathcal{B}(R) \otimes_R \mathcal{B}(R)$.

January 26: On a differential in the spectral sequences needed for the exact sequences

$$0 \rightarrow \overline{HC}_{2n+1}(A) \rightarrow I^{n+1}/[I, I^n] \xrightarrow{d'} H_1(R, I^n) \rightarrow \overline{HC}_{2n}(A) \rightarrow 0,$$

$$0 \rightarrow HC_{2n}(A) \rightarrow HC_0(R/I^{n+1}) \xrightarrow{d'} H_1(R, R/I^n)_\sigma \rightarrow HC_{2n-1}(A) \rightarrow 0.$$

January 27: On maps $(I \overset{!}{\otimes}_R)^{p+1} \rightarrow (I \overset{!}{\otimes}_R)_\sigma^p$.

January 28: Resolution of paradox concerning $\mathcal{B}(R) \otimes_R R[\Delta(1)] \otimes_R \mathcal{B}(R) \rightarrow \mathcal{B}(R)$.

January 29: Horizontal differential $\sigma : \Sigma(I \overset{!}{\otimes}_R)^{p+1} \rightarrow (I \overset{!}{\otimes}_R)_\sigma^p$.

January 30: To find the S -operator on the double complex with columns $\Sigma(I \overset{!}{\otimes}_R)_\lambda^{p+1}$

February 2: The reduced complex $\overline{CC}(R)$ is quasi-isomorphic to $CC(A)/CC(\mathbb{C})$.

February 16: Notes on Walhausen's big theorem, cyclic nerve of a category and Goodwillie's geometric trace map. Also notes on Wodzicki's spectral sequence.

February 19: Reduced cyclic complex of $R \leftarrow I$ where R is unital. Morita invariance of Hochschild and cyclic homology.

February 24: Review of residues, duality and Lefschetz framework.

February 25: On the proof of Atiyah-Bott-Lefschetz formula.

February 26: Atiyah-Bott-Lefschetz formula for a Riemann surface.

February 28: Cartier's residue construction.

February 29: Reconstructing how Tate found his 1-dimensional residue from Cartier's definition.

March 1: On the isomorphism $\text{Ext}_A^1(M, N) = \text{Der}(A, \text{Hom}_k(M, N))/\text{Im Hom}_k(M, N)$.

March 2: Calculations with A -module resolutions, cup products and non-commutative differential forms.

March 3: On Beilinson and Schechtman paper, "Determinant bundles and Virasoro algebras".

March 4: The Virasoro algebra $\hat{\mathcal{A}}_E$ of (X, E) where E is a vector bundle over a curve X .

March 5: The Riemann-Roch theorem on a complete non-singular curve.

March 8: Generalization of the Cartier-Grothendieck residue symbol. Residues over an algebraic curve. Cyclic 1-cocycle associated to an Lefschetz operator on a circle. Tate's method for constructing a residue as presented by Beilinson-Schechtman.

March 10, 11: More on the Beilinson-Schechtman residue construction.

1988-2

Quillen's own index for March 13 - May 5, 1988.

March 13: Beilinson-Schlichtman account of Tate's approach to residues.

March 14, 15: Deriving a formula for the cyclic 2-cycle associated to a square zero extension.

March 16: On Goodwillie's theorem that a nilpotent extension induces an isomorphism on cyclic homology.

March 17: Alternative approaches to Connes homomorphism.

March 18: More on Connes homomorphism. Interpreting $HC_*(A)$ as the universal recipient group for traces of higher orders.

March 19: Stinespring circle of ideas.

March 20: Representing cyclic cocycles using iterated bimodule extensions.

March 25: Quillen's method for producing Hochschild classes.

March 26: Calculations related to Tate's theory concerning a ring C with ideals I_1, I_2 such that $C = I_1 \oplus I_2$.

1988-3

March 27: Using a map $\rho : A \rightarrow B$ with $\rho(1) = 1$ to make $A \oplus A \otimes B \otimes A$ into an algebra.

March 28: Using Stinespring methods to construct odd cocycles attached to an extension.

March 29, 30, 31: More on Tate's theory for $R = I_1 \oplus I_2$ where I_1 and I_2 are ideals. Multiplier algebra.

April 1: Calculations for an ideal I in R where $I^2 = I$.

April 2: Calculations for an algebra R , e an idempotents in R , $F = 2e - 1$ and $\delta : R \rightarrow M$ a derivation where M is an R -module.

April 3: On studying cyclic homology using the cyclic complex applied to DG chain algebras. Higher traces. Program and problems.

April 4, 5: On length 2 chain algebra resolution of $\mathbb{C}[\mathbb{Z}/2]$.

April 6: Calculations in an algebra C with a given subalgebra A and an idempotent.

April 7, 8: Chern-Simons algebra.

April 8: On subshifts of finite type, Markov partitions, Cuntz-Krieger C^* -algebras,

April 9: Return to $C = A * \mathbb{C}[\mathbb{Z}/2]$.

April 10: Calculations and insights connected with a representation of an algebra with an idempotent.

April 11: Details for a letter to Kassels.

1988-4

April 11, 14: Letter to Kassel in which Quillen describes his latest ideas.

April 13: Equivalence between profinite sets and compact, Hausdorff, totally disconnected spaces. Markov partitions.

April 15, 16: Subshift of finite type.

April 17: Poset of Markov partitions and simplices. Cuntz-Krieger algebras.

April 18 - 22: More on Cuntz-Krieger algebras.

April 23: Return to studying cyclic theory of a cochain algebra $C^*(A, B)$ with a 1-cochain $\rho : A \rightarrow B$ such that $d\rho + \rho^2 \in C^2(A, I)$.

April 24: Abstract construction of cyclic cocycles attached to unbounded p -summable Fredholm modules (A, H, D) . Traces on B/I^∞ .

April 28 - May 2: Cuntz-Krieger C^* -algebras.

May 2: Cyclic homology and extensions.

May 3: Deriving exact sequences for $HC_*(\mathcal{A}) = \overline{HC}_*(\mathbb{C} \oplus \mathcal{A})$.

May 4, 5: Using Quillen's Chern-Simons formula for an even Connes homomorphism to prove an index theorem.

1988-5

Quillen's own index for May 6 - August 14 (very faded).

May 6: Review earlier work on cyclic cocycles and left invariant forms on groups of gauge transformations. Understanding the S -operator.

May 7: Restricting cocycles to a direct summand. More on the S -operator. Cyclic cocycles associated to a connection.

May 8: On a super-algebra and an odd involution F such that $[F, \cdot]^2 = 0$. Connes cocycles and two proofs.

May 9,10: The relationship between Connes cocycles and the S -operator. Cyclic cocycles on $C^\infty(M)$.

May 11: Finite dimensional algebras and the S -operator.

May 12, 13: Transgression calculation. Formalism explaining odd Connes cocycles.

May 14: Dilation process. Equivalence of cocycles obtained by dilation. Transgression and super-connection methods.

May 15: Geometric approach to super-connection forms.

May 16: Review of formalism used in constructing Connes cocycles. S -operator. Proof that different Connes cocycles are related by the S -operator.

May 17: The difference between character forms associated to two flat connections on a bundle. Calculations applying super-connection methods to deriving a Chern-Simons form based on a path $d + tA$ of connections.

May 18: BRS-geometry. Witten's Lagrangian for topological QFT using equivariant differential forms for g -manifold.

May 20: Witten's formula for the Chern-Weil homomorphism on a G -manifold with a free G -action.

May 21: Example of gauge fixing and associated formulas.

May 22: An integral formula for Witten's map. Atiyah's idea.

May 24: Universal relations in the double cochain algebra $C^*(A, \Omega_A^*)$ with canonical element $\theta \in C^1(A, \Omega_A^0)$. Review of the Dubois-Violette, Talon, Viallet results. Relation between the BRS algebra of DVTV and anomalies.

May 25: Chern-Simons forms - components of $\int_0^1 dt \operatorname{tr}(\theta e^{td\theta + (t^2 - t)\theta^2})$.

May 26: Splitting of the Chern-Simons transgression form and cyclic formalism. Review of Connes cocycles and also Quillen's method.

May 27, 28, 29: The S -operation for odd cocycles belonging to an extension.

May 30: More on the S -operations.

May 31: Methods for producing left invariant forms on a group of gauge transformations and cyclic cocycles.

1988-6

June 1: Discussion of anomalies. Using material in a paper of Dubois-Violette, Talon and Viallet to produce cyclic cocycles associates to a connection.

June 2: On $\Omega(\mathcal{G} \times M)^{\mathcal{G}} = \Omega(\mathcal{G})^{\mathcal{G}} \otimes \Omega(M)$, where P is a G bundle over M , $\mathcal{G} = \text{Aut}(P)$.

June 3, 4: On Bismut forms for a manifold with a S^1 action. Cyclic theory using the bicomplex $(A \times \overline{A}^{\otimes n}, b, B)$ for a unital algebra A . Constructing homotopy equivalences between the space of pairs of projectors e, e' on a Hilbert space with $e - e'$ compact and $\text{Im } e$ is of infinite dimension and codimension, and the restricted Grassmannian.

June 5, 6: Linking different cocycles belonging to an extension by the S -operator.

June 7: Connes approach to the S -operator.

June 10, 11: The isomorphism of $A * A$ and Ω_A^* . Classical limit filtration construction $\oplus_{p \geq 0} h^p F_q A$ in the context of Clifford and exterior algebras, Weyl and polynomial algebras.

June 12: Construction of Connes cocycles. Connes entire cocycles paper.

June 14: Account of conversation with Stora about gauge fixing and ghosts.

June 15 The BRS algebra and $H^*(\mathcal{G})$.

June 20, 22: Calculations using the left $T(A)/(1 - \rho(1_A))$ module structure on Ω_A^* .

June 23: Understanding BRS (examples from Itzykson).

June 24: Finding an additive description of $C = \begin{pmatrix} eCe & eC\bar{e} \\ \bar{e}Ce & \bar{e}C\bar{e} \end{pmatrix}$

June 26: Definitive picture of the relationship between $A * A$ and Ω_A .

1988-7

June 27: A good construction of Connes cocycles.

June 28: On the exact sequence

$$0 \rightarrow \overline{HC}_{2n-1}(A) \rightarrow I^n/[I, I^{n-1}] \rightarrow I^{n-1} \otimes_B \Omega_B^1 \otimes_B .$$

June 30, July 1: On finding a simple analytical expression for the index given by pairing a Dirac operator with a gauge transformation.

July 4: For (E, ∇, ρ) over a manifold M , on finding a Dirac operator analogue of the associated odd character form.

July 5: Pairing $\alpha \in KK(C(M), \mathbb{C})$ with $KK(\mathbb{C}, C(M)) \rightarrow KK(\mathbb{C}, \mathbb{C})$.

July 7: Wave packet transform on \mathbb{R} .

July 8: Return to index theory on the circle.

July 10, 11, 12: More index theory on the circle and analysis of an approach working over the cotangent bundle.

July 13: Return to associating an index to an odd Dirac operator and a gauge transformation.

1988-8

July 15: On reconciling approaches in cyclic theory given by index theory and Lie algebra cohomology of matrices.

July 16: Connecting the index function and the η function.

July 18: Proof that Connes cocycles are connected by the S -operator.

July 19: Arguments involving the deformation

$$\tilde{F} = \frac{1}{\sqrt{1+t^2}} \begin{pmatrix} F & 0 \\ 0 & -F \end{pmatrix} + t \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

July 20, 21: Review of formulas.

July 24: More on $\tilde{F} = \frac{1}{\sqrt{1+t^2}} \begin{pmatrix} F & t \\ t & -F \end{pmatrix}$. Review of some steps of the proof of the S -relation for Connes cocycles.

July 26: Finding the n th order path algebra. On the S -operator relation between cyclic cocycles $\text{tr}(\delta\rho + \rho^2)^n$.

July 27, 28: Calculations for $A = R/I$ where R is a free algebra. Resulting problems.

July 29: Proof that free algebras have trivial cyclic homology.

August 1 - 4: On constructing a natural map $CC(R \leftarrow I) \leftarrow CC(A)$ for $A = R/I$, starting from a lifting $\rho : A \rightarrow R$. General discussion of DG algebras and coalgebras.

1988-9

Quillen's own index for August 21 - November 11, 1988

August 21: DG algebras, DG coalgebras and twisting cochains.

August 23: Cyclic homology of $k[G]$ where G is a free group.

September 1, 2: Computing the $U(V)$ equivariant forms representing characteristic forms on $\text{Grass}(V)$.

September 5: Leray spectral sequence for a principal bundle. The spectral sequence with $E_1^{pq} = H^q(\mathfrak{h}, \bigwedge^p(\mathfrak{G}/\mathfrak{h})^*) \Rightarrow H^{p+q}(\mathfrak{G})$ where \mathfrak{h} is a subalgebra of the Lie algebra \mathfrak{G} . The triangle of complexes $CC(\mathbb{C}) \rightarrow CC(A) \rightarrow \overline{CC}(A)$.

September 6: Showing Connes homomorphisms $HC_{n-1}(A) \rightarrow I^n/[I, I^{n-1}]$ are compatible with the S -operation.

September 7: Formalism explaining why character forms are linked by the S -operator.

September 8: On the double complex $CC(R \leftarrow I)$ of the I -adic filtration of $\Omega_R^1 \otimes_R$. Understanding the maps $\Sigma(I \overset{L}{\otimes}_R)_{\sigma}^{n+1} \rightarrow (I \overset{L}{\otimes}_R)_{\sigma}^n$ from the viewpoint of derived categories.

September 12: Formula for the natural map from $CC(A)$ to the Connes (b, B) category.

September 13: To find a formula for the S -operator $\overline{HC}_{2n}(A) \xrightarrow{S^n} \overline{HC}_0(A) = \overline{A}/[A, A]$ when A is unital.

September 15: Understanding the cyclic theory of $C^*(A, R) = \text{Hom}_K(\text{Bar}(A), R)$.

September 16: On the character forms $\text{tr} \left(\frac{\gamma^n}{n!} \right) \in R^n/[R, R]$ where (R^*, δ) is a DGA, $\rho \in R^1$ and $\gamma = \delta\rho + \rho^2$.

September 17: Calculations for $R = A^* \otimes B$ where A^* is a DGA and B an algebra with ideal I .

September 18: On $\tilde{\Omega}_A = \Omega_A^1 \otimes_A$.

September 20: Proving the S -relation between Connes cocycles attached to a Fredholm module.

September 22: On the S -relation between the canonical maps from the cyclic complex to De Rham cohomology.

September 23: Clarifying signs encountered in cyclic homology calculations.

September 28: On the bicomplex made from the reduced Hochschild complex with the (b, B) operators.

September 30: On producing bivariant classes in $R\mathrm{Hom}_S(CC(A), CC(B))$.

October 1: Finding a coalgebra structure on $\mathcal{B}^N(A)$, the standard normalized bar resolution of A as an A -bimodule.

October 2: F -modules.

October 3: On interpreting the Hochschild complex. Left invariant forms on gauge transformation groups.

October 4: More invariant theory for gauge transformation groups. Linking up Chern-Simons homomorphisms giving even Connes homomorphisms $HC_{2n}(A) \rightarrow HC_0(R/I^{n+1})$ to Connes graded F -cycles.

1988-10

October 5: K -pairings. Calculations for idempotents e, e' over R which are congruent modulo an ideal I . Linking up Connes approach to even cyclic cocycles with Quillen's approach based on Chern-Simons forms.

October 6: On the DGA of (bar)cochains with values in the Cuntz algebras $A \ast A$ and $(A \ast A) \tilde{\otimes} \mathbb{C}[F]$.

October 7: On obtaining Jaffe-Lesmiewski cocycles,

October 12: Review of the cyclic formalism and Hochschild cohomology.

October 13: The map $\Sigma \mathcal{G} \rightarrow U$ where \mathcal{G} is a group of gauge transformations acting on a space of connections. The map induced by associating to a connection the Cayley transform of the Dirac operator. Constructing odd equivariant forms for the conjugation action of $U(n)$ on itself.

October 14: More on equivariant functions for $U(H)$ acting by conjugation on $U^\infty(U, -1)$.

October 19: Comments on a non-unital A -bimodule algebra, on a \ast -algebra acting on a Hilbert space, on Hochschild cohomology, on Lie algebra cochains and on invariant theory.

October 20: Notes on an equivariant vector bundle over a Lie group, on applications to the Fredholm module (A, H, F) and on Lie algebra cochains.

October 21: Lie algebra interpretation of Hochschild cochains which come from a super-trace on $A \ast A$.

October 22: Review of interests.

October 26: Review of the link between the Weil algebra and the Narasimhan-Ramanan theorem viewpoints. Equivariant forms on $U(n)$ with the conjugation action.

October 27: On understanding the JLO big cocycle by considering the cochain algebra $C(A, \mathcal{L}(H))[\sigma]$ and using the super-connection $\delta + \theta + X\sigma$.

October 28: Establishing the S -relation between cyclic cocycles. Comment on $\Omega_R^1 \otimes_R$.

November 3: The exactness of $R^{\otimes^4} \xrightarrow{b'} R^{\otimes^3} \xrightarrow{p} R \otimes V \otimes R \rightarrow 0$.

November 4: A simple construction of the exact sequence $0 \rightarrow c \otimes A \xrightarrow{i} C^{\otimes 2} \rightarrow C^{\otimes 3}$ by duality and exactness of $0 \rightarrow A[1] \otimes C \xrightarrow{\tilde{i}} C \otimes \overline{C} \rightarrow C \otimes \overline{C}^{\otimes 2}$.

November 9: Calculations in the unbounded Fredholm situation.

November 11: Big cochains associated to a bundle with a connection and a cycle.

Lecture Notes

Notes for some lectures on cyclic cohomology.

Contents 1989

1989-1

Quillen's own index for January 12 - June 11, 1989.

January 12: Linear functionals on $\text{Tr}(A) = (A * A)^+$.

January 13: Traces on $(A * A)^+$ and even supertraces on $A * A$.

January 14: Traces on GNS. Pairing with $K_0(A)$. Lifting representations.

January 16: The bar construction for a DGA.

February 4: Formalism concerning $M_r(A)$ and A , and map $B(A \otimes S) \rightarrow B(A) \otimes S$ (specialising to $S = M_r(k)$).

February 10: Manipulations connected with $X \otimes M$ where M is a left R module and X is a right C -module, C is a coalgebra and R an algebra.

February 22: Filtering $B(I \hookrightarrow R)$.

March 8,9: On approaches to the Novikov conjecture.

March 13: Canonical additive isomorphism $A * A \simeq \Omega_A$.

March 14, 15: The GNS construction and map $A \otimes B \hookrightarrow \text{Hom}(A, B)$, $a \otimes b \mapsto (\alpha \mapsto \rho(\alpha)b)$. Supertraces on $A * A$ and traces on $(A * A)^+$.

March 25, 27: Understanding the GNS-NR ideas and Connes $b + B$. Connecting the NR setting with the complex Connes supertrace on $A * A$. I-adic filtered version of the periodic complex for R as a quotient of $CC(I \rightarrow R)$

March 29, 30, 31; Proof of exactness of

$$0 \rightarrow HC_{2n-1}(A) \rightarrow I^n/[I, I^{n-1}] \xrightarrow{d} \Omega_R^1 \otimes_R (R/I^n) \otimes_R$$

and

$$0 \rightarrow HC_{2n}(A) \rightarrow I^n/[I, I^{n-1}] \xrightarrow{d} \Omega_R^1 \otimes_R (R/I^n) \otimes_R .$$

April 2: List of ideas to work on.

April 4, 5: Extension process from odd cocycles to traces. Calculations for free algebras.

April 6-10: Comments on proofs of theorems for extensions using the Chern-Simons cocycle with values in a quotient complex.

April 12: Linking Connes approach (using traces on the Cuntz algebra) with JLO.

April 15: Comments on variation maps.

1989-2

April 16: Summary of some K -theory and index theory associated to Fredholm modules.

April 17, 18: The JLO cocycle.

April 17, 18: The coset of bounded perturbations of an unbounded skew-adjoint operator.

April 28: Connes process for passing from group cohomology of a hyperbolic group Γ to entire cyclic cocycles on the group algebra $\mathbb{C}\Gamma$.

April 30: BRS analysis of anomalies and counter terms.

May 1: Characterizing functions of the form $f(x; v_1, v_2, \dots, v_n) = \partial_{v_1} \partial_{v_2} \dots \partial_{v_n} F(x)$.

May 4: Discussion on the situation of a derivation $D : A \rightarrow M$ inducing $HC_n(A) \xrightarrow{D_*} HC_n(A, M)$ factorizing through $HC_n(A) \xrightarrow{B} H_{n+1}(A, \hat{A})$. On Hochschild cocycles $\psi(a_0, a_1, \dots, a_n), \phi(a_1, a_2, \dots, a_n)$ where $b'(\phi) = (1 - \lambda)\psi$ and $b\psi = 0$.

May 10: On the exact sequence

$$0 \rightarrow I/I^2 \rightarrow A \otimes_R \Omega_R^1 \otimes_R A \rightarrow \Omega^1 2_A \rightarrow 0$$

associated to an extension $A = R/I$.

May 15, 16: Morita invariance. Inner automorphisms act trivially on cyclic homology.

May 17: Unital and non-unital algebras and their bimodules.

May 19, 20: Algebra extensions and bimodules extensions. Canonical model for $\overline{HC}^1(A)$.

May 22: Review of recent calculations.

May 23: Calculations for the S -operation based on non-commutative differential forms and Chern-Simons.

June 5, 6: Non-commutative versions of the Weil algebras and the BRS algebra.

June 11: Calculations of Connes S -operator using differential forms and Chern-Simons cocycles.

Quillen's own index June 21 - August 28, 1989.

June 21: Construction of Chern-Simons forms and S -relations.

July 1, 2: Notes on Stasheff's papers, particularly his discussion of BRS.

July 6-11: Stasheff's ideas on perturbations in a differential graded algebra.

July 18: Milgram's version of the free loop space BG .

July 25: Unbounded Fredholm modules and entire cyclic theory.

1989-3

July 27, 28: Spectral flow and the subvariety of operators having 0 as an eigenvalue. The geometry of the subvariety of singular operators.

July 29: Calculations for A_0, A_1 bounded, invertible, self-adjoint operators where $A_0 - A_1$ is compact; also for D_0, D_1 unbounded, self-adjoint operators where the resolvents are compact and $D_0 - D_1$ is bounded.

July 30, 31, August 2: For A an invertible, self-adjoint operator, calculations related to the behaviour as A varies of the projection P onto the part where $A > 0$. Calculation of relative index for A, A' invertible, self-adjoint operators with $A - A'$ bounded and A^{-1}, A'^{-1} compact and their projections onto their positive parts. Calculations on U_n for the function $g \mapsto \det(\phi(g))$ where ϕ is a certain degree 1 map. Link between the singular divisor and 1-form representing the determinant class. Index associated with two idempotents e, e' where $e - e'$ is compact.

August 2: The \mathcal{L}^p -case when the determinant $\det\left(\frac{X}{t+X}\right)$ exists and renormalized determinants.

August 3: Link with η invariant.

August 4, 6: Discussion of the topology and geometry of the determinant class in $H^1(U_n, \mathbb{Z})$ and the subvariety $Z = \{g \in U_n | \det(g - 1) = 0\}$.

August 8: Further discussion of an invertible, self-adjoint operator A such that A^{-1} is compact and a deformation $A + B$ where B is bounded.

August 9, 10: Discussion of the index formula $\text{Index}(u) = \text{tr}((1 - qp)^n - (1 - pq)^n)$ where u is invertible modulo I, p, q lift u, u^{-1} , respectively, and the trace is defined on I^n . Connes-Moscovici-Bismut

techniques used on cyclic cocycles associated to Dirac operators as transgressed Chern-Simons forms.

August 12: Generalising from $F^2 = 1$ to $F^2 - 1$ compact.

August 15, 16: Understanding index theory and cyclic cocycles where $F^2 = 1$ is replaced by $F^2 - 1$ lying in a smooth ideal.

August 17, 18: Natural notion of homotopy between morphisms $\theta_0, \theta_1 : R \rightarrow S$ of DG algebras both satisfying $\delta\theta_i + \theta_i^2 = d\theta_i = 0$. Review of perturbation theory of Guggenheim et al.

August 18, 19: Calculations for $A \rightarrow R \supset I$, $z \in R$, $[z, A]$, $1 - z^2 \in I$.

August 20: Review of discussion of homotopy for DGA maps. Application to cyclic complexes.

August 21, 24: Perturbation theory applied to contracting morphisms. On the cochain

$$-\frac{1}{2}\{\theta P^{(2n-1)}([\frac{z}{2}, \theta] - \theta(\frac{1-z^2}{4})\theta)\}$$

August 27: Comments on Weiner measure on a Riemannian manifold.

August 28: Discussion of some of Ezra Getzler's ideas. Witten currents.

1989-4

Quillen's own index September 3 - December 19, 1989.

September 4: Cyclic cocycles for a unital algebra R generated by a non-unital subalgebra A and an element z .

September 4, 6: Traces and supertraces on $Q \hat{\otimes} C_1$ where Q is a superalgebra. Unifying the graded and ungraded versions for $\tilde{A} * \mathbb{C}[z]$ where z is odd and \tilde{A} is even.

September 10: On the p -summable Fredholm module where A, F acting on H with $[F, a] \in \mathcal{L}^p(H)$ for all $a \in A$.

September 16, 17: Using Tomita-Takesaki-KMS theory to find an algebra R associated to A such that traces on R give cyclic cocycles on A . Connes theory.

September 23: On a square zero extension with lifting: $0 \rightarrow M \rightarrow A \xrightarrow{\ell} 0$.

September 24, 25: Using $\rho \in R \otimes S$ to produce elements $e^{\tilde{w}} \in R \otimes S + \Omega_{R, \mathfrak{h}}^1 \otimes \Omega_{S, \mathfrak{h}}^1$, $\partial \tilde{\rho} e^{\tilde{w}} \in \Omega_{R, \mathfrak{h}}^1 \otimes S + R \otimes \Omega_{S, \mathfrak{h}}^1$.

September 26: Calculations involving $k[\epsilon] = k[X]/(X^2)$, where A and R are unital algebras and $A \rightarrow R \otimes k[\epsilon]$ is an algebra homomorphism.

September 27: On two-parameter families of homomorphisms $u_{st} : A \rightarrow R$ and a traces τ on R : a homotopy of homotopies and traces.

1989-5

September 28: Notes concerning $R \otimes S$ and the tensor product of mixed complexes $(\Omega_R^1 \rightrightarrows R) \otimes (\Omega_S^1 \rightrightarrows S)$.

October 14: On p -summable Fredholm modules, traces and supertraces. Pseudo-differential operators and non-commutative residue over a torus \mathbb{R}^r/Γ as a trace on the cross-product of the group algebra of Γ with constant coefficient pseudo-differential operators.

October 15; Two versions of the Index theorem for \mathbb{R}^r/Γ and relation to non-commutative residue.

October 17, 18, 21: Weyl quantization map and twisted convolution algebras.

October 23: Remarks on $\text{GNS}(\rho)$.

October 25, 28, 29: Interpreting d on ΩA when $\Omega A = \text{gr}(QA)$.

October 30, November 2: On the normalized Hochschild complex of A with coefficients in A and the Karoubi operation: $\kappa(a_0 da_1 \dots da_n) \mapsto (-1)^{n-1} da_n a_0 da_1 \dots da_{n-1}$

November 3: Review of discussion with Jacek Brodzki on the Goodwillie result: $L_D S - S L_D = 0$.

November 4: Variation maps.

November 5-17: Homotopy for QA .

November 19: Karoub's map κ on traces.

November 26: The 1-form $\int_{M^{2n+1}} (\delta A F^n)$.

November 27: Ideas listed for future reference.

December 2: Calculations checking homotopy for QA . Cuntz's approach to supertraces on $\Omega^1 A$ and Quillen's approach to QLQ .

December 3-7: The operator $1 - \frac{1}{2}bs$ on normalised Hochschild cochains. Cuntz's homotopy is different from Quillen's.

December 10: GNS when $A = k[F]$.

December 19: JLO cocycles and parity issues. Geometric interpretation of Hochschild cochains.

Contents 1990

1990-1

Quillen's own index for January 22 - April 11, 1990.

January 22: Notes on BRS cohomology.

January 25: An analogy between the $(b, S, 1 - \kappa, B)$ operators and the (d, i_X, L_X, Pi_X) operators for manifolds with a circle action.

February 2: An analogue of the S operation. Transgression.

February 3: Lundell's construction deforming $S^2 \wedge U_n \rightarrow U_{2n}$ to a map $S^2 \wedge U_n \rightarrow U_{n+1}$.

February 4: Two methods for defining classes $cs_{2n-1} \in H^{2n-1}(P)$, $2n > \dim B$, where $P \rightarrow B$ is a principal U_n -bundle over B .

February 6: Review of Bott map.

February 10: On $U_N/\Delta_n S^1 \times U_{N-n}$. Chern-Simons forms on a U_n -bundle. Variation maps.

February 12,15,16: Notes about Feigin-Tsygan on Lie algebra cohomology and Riemann-Roch.

February 20: Lecture on Lie algebra cohomology.

February 21-28: Leray spectral sequence for the principal bundle $G \rightarrow P \xrightarrow{\pi} B$. Spectral sequence arising from the bigraded differential algebra $\Omega(P) \otimes \mathfrak{g}_\chi^* \otimes S\mathfrak{g}_\phi^*$. Bott's spectral sequence where $E_2 = H_{\text{diff}}^*(G, S\mathfrak{g}^*) \Rightarrow H^1(BG)$. Review of Leray and Bott spectral sequences.

1990-2

March 11: More on Karoubi's κ operator.

March 17: Formulas for a circle action on a manifolds and a discrete analogue in cyclic theory.

March 18: More on Karoubi's κ operator and an S operator.

March 20: Formulas connected with the periodic complex $\cdots \rightarrow \overline{Q} \xrightarrow{d} (\Omega^1 Q)_\natural \xrightarrow{\beta} \overline{Q} \rightarrow \cdots$. Bismut's construction for an S^1 -manifold. Explicit calculation of the space of invariant cochains.

March 21: Questions and ideas related to the March 20 work.

March 23: On the exact sequence $0 \rightarrow s\mathcal{C}^{n+1} \rightarrow \mathcal{C}^n \xrightarrow{\tilde{s}} s\mathcal{C}^n \rightarrow 0$. On $\rho_A \rightarrow k$ as a connection and explicit formulas for S .

March 26: Karoubi's $\tilde{\kappa}$ on $\Omega^1 A$.

March 27: More formulas related to $\Omega^n = A \otimes \overline{A}^n$.

March 28: Analysis of the Goodwillie theorems about derivations.

March 29,30: More on derivations.

April 2: A theorem on exact sequences $0 \rightarrow X \xrightarrow{i} E \xrightarrow{p} Y \rightarrow 0$.

April 3,4,6: More on Goodwillie-Rinehart.

April 7: On the map $b' : A \times \overline{A}^{\otimes n} \otimes A \rightarrow A \otimes \overline{A}^{\otimes(n-1)} \otimes A$. A contracting homotopy for the Hochschild complex in degrees > 1 .

1990-3 April 11: Feit conference. Outline of Serre's lectures on Galois groups and cohomology.

April 12,13: Derivations i_D^* and L_D induces by a derivation on ΩA . $I_D^2 = [B, [b, H]]$.

April 14: Rinehart's formulas.

April 16: On a representations of DG Lie algebra with basis L , i .

April 25: Facts about mixed complexes.

May 8: I -adic filtrations.

May 21, 27: Defining L_D , i_D^* on ΩA .

May 8-June 5: Homotopy for $X(A)$.

June 8: Continuity of the homotopy with respect to the I -adic filtration.

June 10: Some ideas.

June 15,21: New idea using the mixed complex (Ω, b, B) .

1990-4

July 3: On the super-symmetric time evolution operator $e^{\tau X + t X^2}$ where τ is the Grassmann variable and t an ordinary variable.

July 5-9: Cyclic homology of A where $\Omega^1 A$ is projective, $A = B/I$ with I nilpotent.

July 10: Review of earlier work on tensor products.

July 12-14: Coalgebras in the category of A -bimodules. Bimodule version of the bar construction.

July 16-18: Review of square zero extensions of algebras. Hochschild homology and Connes exact sequence in the case of a presentation $A = R/I$ with R smooth.

July 19, 20: On the $\mathbb{Z}/2$ complex

$$R/I^{m+1} + [R, I^m] \xrightleftharpoons[d]{b} (\Omega^1 R/I^m \Omega^1 R)_{\natural}.$$

July 27: $\mathbb{Z}/2$ graded complexes $X(Q)$ and $X(Q^s)$ associated to $Q = QA$ considered either as an algebra or a superalgebra.

August 9,11: On the superalgebra $A * k[F]$.

1990-5

August 30-September 6: Differential algebra calculations for subalgebras S and Q such that $S \otimes Q \cong E$. Relative theory for a map $S \rightarrow A$ of algebras with relative constructions $R(A; S)$, $Q(A; S)$, $A *_S A$, $\Omega(A; S)$,

September 12: Proof that $\Omega^1(R; A) \simeq R \otimes_A M \otimes_A R$ where $R = T_A(M)$ and M is an A -bimodule.

September 19, 20: Fredholm modules over A and calculations with $EA = A * \mathbb{C}[F] = (QA) \times \mathbb{Z}/2$.

September 28,29: Rough notes on $R = S \otimes Q$.

October 9: On $\Omega^1 R$.

October 11: On $R^e = R \otimes R^o$.

October 12: Derivations and $R \otimes R^o$.

November 1: Higher homotopies for traces. Summary of ideas for future reference: Kunneth theorem; deformation theory of $P\Omega(A)$; maps on periodic cyclic theory and asymptotic maps; using $X(A)$ to establish periodic cyclic homology; $(P\Omega, b, \Omega)$ gives cyclic homology and the stabilization mystery behind K -theory.

November 3: Calculation with the I -adic filtration on $R \otimes S$ where I is the ideal generated by $[R, S]$.

November 4: Polynomial families of lifting homomorphisms $A \rightarrow R$ where $A = R/I$ and $I^{m+1} = 0$.

November 8: Square zero extensions.

November 10: Traces and homology. List of ideas to develop later: Index theory on a torus; Morita type maps; homotopy.

November 11,12: Natural homomorphism: $K_1^{alg}(A) \rightarrow \text{Ker}\{\Omega^1 A_{\natural} \rightarrow \Omega^0 A_{\natural,\kappa}\}$ given by $g \mapsto \text{tr}(g^{-1}dg)$.

November 15: Fedosov's proof of the Index theorem and Connes tangent groupoid.

1990-6

November 16: On $X(R) = \varprojlim X(R/I^n)$.

November 17: First order derivation of homomorphisms.

November 18: On the projection $\Omega^1 R \otimes R \rightarrow \Omega^2$ where $\Omega^1 R$ is a projection.

November 20, 21: Adic topological algebra.

November 21: Deformations and Block's theorem.

November 28: On $\hat{R} = \varprojlim X(R/I^n)$. The Hochschild complex $A \otimes_A^{\mathbb{L}}$ in a derived category framework.

November 29,30: Exploiting results from adic filtrations.

December 2: Reduced cyclic homology.

December 3: Why \overline{HC}_n and H_n^{DR} are not Morita invariant.

December 9: Notation for the opposite algebra R^o and the enveloping algebra R^e . Summary of identities for Karoubi's κ operation.

December 12: On transformations of finite order.

December 18: Towards understanding homotopy and restricted homotopy.

December 20: Polynomial families of homomorphisms.

December 22: The B operator on the Hochschild complex associated to $A = R/I$ where $\Omega^1 R$ is projective.

December 25,26,28: More on the B operator on the Hochschild complex.

December 31: To show that the truncated complex $X^n(R; I)$ is invariant under restricted homotopy.

Contents 1991

1991-1

January 3: Connes exact sequence via extensions.

January 5: Gauss-Manin connection for a family of algebras: $A_h = \mathbb{C}_h \otimes_{\mathbb{C}[t]} \mathcal{A}$, $\mathbb{C}_h = \mathbb{C}[t]/(t-h)$ and \mathcal{A} an algebra over $\mathbb{C}[t]$.

January 9,11: Proof of the Krull-Schmidt theorem. New viewpoint on Karoubi's operator.

January 18: Connes tangent groupoid.

January 20,21: Morita aspect of cyclic theory.

January 23,24: Notes on why the mixed complex (Ω, A, b, B) gives cyclic theory of A .

January 26: Notes about the exact sequence associated to $R = T(V)$:

$$0 \rightarrow \Omega_{\mathfrak{h}}^2 \xrightleftharpoons[-b]{-k} \Omega^1 R \xrightleftharpoons[\mathfrak{h}]{h} \Omega^1 R_{\mathfrak{h}} \rightarrow 0,$$

January 28: Connections on right R -modules and exact sequences with splitting.

February 1: First order variation of the X -complex Lie derivative L .

February 3,5: Review extensions of non-commutative algebras A over a commutative ground field S and relative differentials $\Omega^1(A; S)$.

February 6: Constructions with $A = R/I$, A quasi-free, I nilpotent.

February 7: Free products.

February 8: Note for future calculations in $R * S$.

February 9: Linear functions on $\Omega^1(R * S)_{\mathfrak{h}}$.

February 10: Constructing a complex of complexes $X(A) \rightarrow \Omega(M, X(R))$ where $A = R/I$ is quasi-free, I is nilpotent and M is a space of liftings $\theta : A \rightarrow R$.

February 15: On the square zero extension $R * S/I^2$ where $I = \text{Ker}\{R * S \rightarrow R \otimes S\}$.

February 16: Review of the equivalence between square zero extensions of A and bimodule extensions of $\Omega^1 A$.

February 17: On the periodic homology algebra of projective $\dim \leq 2$. Review of algebra or $R \otimes S$.

February 19: Lifting a homomorphism $\theta : A \rightarrow R \otimes S$, where A is quasi-free, to a homomorphism $A \rightarrow R * S/I^2$.

February 20: On $X(RA)$.

February 21: List of ideas for future reference.

February 22: On $\widehat{RA} \cong \widehat{\Omega A^+}$ as algebras and $\widehat{QA} \cong \widehat{\Omega A}$ as superalgebras. Curvature for separable spaces.

February 24: Separable algebras (projective as A -bimodules).

February 28: On Ω_A and ΩA .

March 1: ΩA as a quotient of $X(RA)$.

1991-2

- March 2: Differential algebras and the Fedosov product: $x \circ y = xy + c(-1)^{|x|} dx dy$.
- March 4: Poisson manifolds.
- March 6,7: On the square zero extension R/I^2 where $R = T(V)$, $A = R/I$, $I = ([v_1, v_2] - w(v_1, v_2))$.
- March 8: Quivers.
- March 9: Representing a generator of $HC^2(A)$ by a trace on a square zero extension where A is a Weyl algebra generated by V , $\dim V = 2$.
- March 14,15: On the Fedosov algebra associated to Ω_A^+ .
- March 26,28: More on Poisson structures.
- March 30,31, April 1,2: Review of KMS for Gaussian states on a Weyl algebra.
- April 6: Universal description of R_A for A commutative.

1991-3

- April 7: Square zero algebra extensions $0 \rightarrow M \rightarrow B \rightarrow A \rightarrow 0$ where A is commutative and M is an A -module.
- April 11: Tensor products $X(R) \otimes X(S)$.
- April 12-19: Calculating in $R * S/I_{R,S}$ where $I_{R,S} = (R * S)[R, S](R * S)$.
- April 30: Direct computation of $X(RA)$.
- May 5: Relative theory.
- May 6: Fedosov algebra computations. Computing the maps $K_i A \rightarrow H_i(X(RA)) \rightarrow H_{i \bmod 2}(\Omega_A^\pm)$.
- May 7: Adjoining an identity.
- May 25: On the canonical maps $K_i A \rightarrow HP_i A$, $i = 0, 1$.
- May 27: The universal property of R_A .
- June 12,13: On proving the image of S in A is separable when the homomorphism $S \rightarrow A$ is such that $A \otimes_S A$ is a projective bimodule of A .
- June 14: On finite dimensional quasi-free algebras.
- June 15,17,18: More on $S \rightarrow A$ with S separable.
- June 19: Note on difficulties associated with the fact that homological algebra methods produce maps of cyclic theories that are not obviously related to homomorphisms on the level of R -algebras.

1991-4

- June 22: On the relative situation where A is an algebra under S with surjection $RA \rightarrow R_S A$.
- June 23,24,25: On the case $S = \mathbb{C}[F]$, $F^2 = 1$.
- June 27: Affine spaces and associated calculations.
- June 28: Calculations for $R = T(V)$ equipped with $1_V \in V$.
- June 29: Calculations for $RA \rightarrow R_S A$, $S = \mathbb{C}[e]$.
- July 12,13: Connections on a bimodule.
- July 15: Calculations pertaining to $\widehat{R}A \cong \widehat{\Omega}^+ A$, $A = \mathbb{C}[F]$.
- July 18: Review of a computational proof that for A separable there is a canonical bimodule lifting $A \rightarrow A \otimes A$. Canonical lifting $\Omega^1 A \rightarrow A \otimes \bar{A} \otimes A$.

July 20: Consequences of assuming $M^{\natural} \cong M_{\natural}$ for all bimodules M .

July 23,24,27: Various calculations.

July 30: Quillen's account of a conversation with Graeme Segal about Kontsevich's work on ribbon graphs.

August 16: Ideas worth working on: characterizing a trace on R in terms of a bimodule extension; Picard category of lines associated to Morita self-equivalences of \mathbb{C} .

1991-5

September 15: On mixed complexes. Notation for cyclic theory and mixed complexes.

September 29: On HC^- from the $\mathbb{Z}/2$ -graded viewpoint.

October 3: Differential operators. Universal enveloping algebras and differential operators.

October 10: A minimal A -bimodule resolution of a smooth commutative algebra A .

October 11: Blowup of $(0,0)$ in $\mathbb{R} \times M$. Tubular neighbourhoods for embeddings of non-singular varieties in manifolds.

October 18: Poisson Lie groups (conversation with Weinstein and Lu).

October 26: On cyclic objects. Proving $B\overline{Cyc} \sim BS^1$.

November 1: Ideas from Goodwillie.

November 15: Dan Freed's topological QFT. Note titled **After Tsygan** .

November 16,20: On a derivation of RA .

November 17: Questions about $\varinjlim M_n \mathbb{C}$.

December 9: Two problems.

December 13: Madsen's talk on the cyclotomic trace.

December 14: Gunner Carleson's talk. Madsen's construction.

December 16: End homology theory of Pedersen-Weibel.

December 22: Cyclic theory for unital and non-unital algebras.

Contents 1992

1992-1

February 20: Relating the Connes unnormalized (b, B) approach to cyclic cohomology to the Quillen normalized approach.

February 23: Tate-Farrell cohomology.

February 24: Cyclic modules and their cyclic theory.

February 27, March 5: Using Karoubi's method to establish Morita invariance for entire cyclic cohomology.

March 6: Computing the cohomology of a small category using covariant functors and homology using contravariant functors. On explaining negative and periodic cyclic homology via analogy with Tate-Farrell.

March 8: Simplicial objects. Formulas for Homological Perturbation Theory (HPT). Homology and cohomology of small categories. Ideas and problems.

March 9: On $T_A(E)$ where $E \rightarrow A$ is a surjection of A -bimodules.

March 10: Map for $K_1 A$ into a negative cyclic homology group.

March 11: Poincaré and Alexander duality at the simplicial level.

March 13: Review of Weil's proof of the de Rham Theorem and the related proof of Poincaré duality.

March 14: Poincaré duality on a combinatorial level. Poincaré duality on posets.

March 15: Summary of formulas for homology and cohomology of posets. Aspects of Grothendieck's theory.

March 16: Proving $H^i(X, F) = H_{n-i}(X, F)$ when F is a covariant functor on a suitable n -dimensional finite poset.

March 17, 18: More on Poincaré duality at a combinatorial level.

March 20: Combinatorial analogue of the cohomology class of the diagonal.

March 25: Note about finite simplicial complexes and their posets of simplices.

March 26: Organizing the various aspects of duality.

March 30: Analysis of duality.

March 31, April 1: More on duality for finite posets.

April 2, 4: Return to $H^n(X \times X^{\text{op}}, \mathbb{C}_{U_\Delta})$ where $U_\Delta = \{(x, y) \in X \times X^{\text{op}} | x \geq y\}$. Ideas to develop further.

1992-2

April 23: Relationships between different models of cyclic theory.

April 24, 25: On minimal free objects.

May 8: Summarizing and organizing recent work starting with the calculation of cyclic theory for universal enveloping algebras and group algebras.

May 12, 13, 14: On Burghelia's theorem: let G be a finite discrete group then the groupoid of G -torsors over S^1 is equivalent to the groupoid given by G acting on itself by conjugation.

May 16: A group homology class lifts canonically to a cyclic homology class.

May 17: Statement of a problem regarding the S^1 action on the free loop space of G .

May 23: G -torsors over a cyclic graph. The category of cyclic graphs has the homotopy type of BS^1 .

May 25: More on cyclic theory.

May 26: More on HPT.

May 27: A projective resolution of an algebra.

May 29,30,31: A constructive version of the fundamental lemma of homological algebra using connections.

June 1-6: On $T_A(A \otimes A)$. Homological algebra of G -bundles.

June 7,8,9: A projective bimodule resolution P of A yields a functor $X \mapsto P \otimes_A X$. Application to Graeme Segal's category of cyclic graphs.

1992-3

June 10,11: More on G -torsors and cyclic graphs.

June 12,13: Review of recent work. More algebraic topology of groupoids and cyclic graphs.

June 16: Review of equivalence of categories.

June 17: Return to cyclic groupoids and torsors over cyclic graphs.

June 18,19: Some category theory.

June 21,23: Some category theory with examples of HPT (Homological Perturbation Theory).

June 27, July 1: Character map from algebraic K -theory to negative cyclic theory and $H_*(G) \rightarrow HC^-(A)$. Comments of cyclic theory of the universal algebra of a Lie algebra.

July 3: On Teleman's way of deforming the Hochschild complex to something like the infinite jets on the diagonal.

July 4: On the DGA, $T_A(A \otimes A)$.

July 9: Linking the tower $\mathcal{X}^q(R, I)$, $A = R/I$, to the Jones-Kassel bivariate cyclic cohomology.

July 10: on the $\mathbb{Z}/2$ graded complex M equipped with a filtration $0 = F_{-1}M \subset F_0M \subset \dots$, $M = \cup F_n M$ such that $F_n M / F_{n-1} M$ has zero homology in parity n .

July 13: Things learnt from studying bivariate cyclic homology and continuing the work of 10 July.

July 25: On homotopy equivalence of complexes.

July 27: Technical addition to HPT.

August 8: On towers of complexes and HPT.

November 22,30: Derived categories. On the derived category \mathcal{C}_Λ of mixed complexes.

December 6: On twisting cochains between a DG coalgebra and a DG algebra.

December 7: Adjunction formula.

December 8: On $M \mapsto \text{Hom}_\tau(\mathbb{C}[S], M)$ and $M \mapsto B(\Lambda) \otimes_\tau M$.

December 12: Consequences of asserting that $HC^k(P, Q) \cong HC^k(X, Y)$ where P and Q are DG S -modules and $X = \alpha P$, $Y = \alpha Q$ associated towers.

December 21: Note on towers.

December 25: Note on patterns and impressions gained from recent calculations.

Contents 1993

1993-1

January 2: Relating cohomology groups of the mapping complex $H^k(\text{Hom}(C, C'))$, with cup product, to bivariant groups defined from H^0 , with cup product. Scinded fibred categories.

January 14: Observations coming from work on bivariant cyclic cohomology $HC^*(M, M')$, with cup product.

January 18: List of ideas for a future program of constructing homotopy equivalences using connections.

February 9: Operators T satisfying a polynomial relation $(fg)(T) = 0$ and contracting homotopies.

March 18: Note on the structure on $X(R)$: $\mu(x \otimes y) = xy$, $\nu(x \otimes y) = \natural(xdy)$ such that $b\nu(x \otimes y) = \mu(x \otimes y) - \mu(y \otimes x)$, $d\mu(x \otimes y) = \nu(x \otimes y) - \nu(y \otimes x)$.

April 9,10: On Karoubi's operator on unnormalized cochains. The normalization theorem for the Hochschild complex and contracting homotopies.

April 15,17: About Connes B_0 (revision of part 2 of a paper). Equivalence of the Getzler-Szenes and Connes definitions for entire cyclic cohomology.

May 7: Link between simplicial normalization and special contractions in the sense of HPT.

May 8: Representing classes in $\text{Ext}_A^n(M, N) = H^*(A, \text{Hom}(M, N))$ by Hochschild cocycles.

May 14,15: Cuntz's application to Nistor's bivariant Chern character.

May 18: Calculating $\text{gr}_I R = \bigoplus_p I^p / I^{p+1}$ where $R = A * \mathbb{C}[h]$, $I = Rh^2R$.

May 19,20: Review $R = A * \mathbb{C}[D]$, $D = 1 \otimes 1$ and $R = T_A(A \otimes A)$.

May 21,22: On a mixed complex M such that $H^b H^B M = 0$.

May 23: On the complex (C, b') with contraction h .

May 25: Calculations for an module M with submodules K, L such that $K \cap L = 0$.

May 26: Review mixed complexes satisfying Connes's Lemma.

May 28,30: Nistor application.

May 31: Calculations for (R, I) and (S, J) and homomorphisms $R \xrightarrow{p^t} S[[t]]$.

June 4: On homomorphisms $A \xrightarrow[\theta']{\theta} L$ congruent modulo ideal $J \subset L$.

June 5: Lemma: Let $I \subset R$ be an ideal. The canonical map $X(R) \rightarrow S_{\natural} \otimes X(R)$ carries $F_{S \otimes I}^p X(S \otimes R)$ into $S_{\natural} \otimes F_I^p X(R)$.

June 6: On maps $A \xrightarrow[\theta']{\theta} L \otimes B$ congruent modulo $J \otimes B$ with $S = \bigoplus_{h \geq 0} t^h J^h \subset L[t]$.

June 7: On algebras S, T and the canonical map $\alpha : X(S \otimes T) \rightarrow S_{\natural} \otimes X(T)$.

1993-2

June 7: Construction of Nistor's bivariant Chern Character.

June 16: Calculations for the standard normalized and unnormalized resolution of the A -bimodule A .

June 17: On the DG algebra $R = A * \mathbb{C}[D]$.

June 22: Reconciling the approaches of Quillen and Nistor (used by Cuntz).

June 25: On the category of filtered vector spaces.

June 26: More on reconciling the Quillen and Cuntz approaches.

June 30: More on Nistor and reconciling approaches to the bivariant Chern character.

July 1: Review progress on Cuntz's version of Nistor's bivariant Chern character.

July 3,8: On Quillen's construction of the bivariant Chern character associated to the quasi-homomorphism A
 $\text{rightarrow} L \otimes B$ congruents modulo $J \otimes B$.

July 20: Relation between $\Omega A = T_A(\Omega^1(A))$ and $T_A(A \otimes A)$ plus a note added on October 15.

August 22: On S versus L^t .

September 21: On HPT. Diluting a contraction to a special contraction.

September 22,23: Mapping cylinder and Serre construction for $X \rightarrow Y$, a map of complexes.

September 24: Perturbed contractions: $h \frac{1}{1-\theta h}, h \frac{1}{1-[\theta, h]}, \frac{1}{1-[\theta, h]} h$.

September 25: Possible contractions on $F(f) = X \oplus Y[-1]$ with $\begin{pmatrix} d & 0 \\ f & -d \end{pmatrix}$. Formulas related to composition.

September 26,27: Note on formulas of HPT.

September 28: Calculations on a complex

$$\cdots \xrightarrow{\partial} M_1 \xrightarrow{\overset{h}{\curvearrowright} \partial} M_0 \xrightarrow{\overset{h}{\curvearrowright} \partial} M_{-1} \rightarrow \cdots$$

of A modules with contracting homotopy h .

September 30: Constructions for \mathcal{C}_A , the category of DG A -modules (bounded below) and the corresponding homotopy category $H_0(\mathcal{C}_A)$.

October 3,4,5: On an augmented DG algebra and its bar construction.

October 7: On the B operator.

October 8,10,11: On $\text{Cone}(E \xrightarrow{\eta} A)$ where A is an algebra, E a complex of bimodules and η a module map.

October 15: On making an S -module out of $\mathcal{B} \otimes (E \otimes_A)$.

October 17: GNS construction result: $\Gamma(A \xrightarrow{\text{in}, \rho} RA * C) = A * \tilde{C}$.

October 27: Cartan-Eilenberg flatness using linear equations.

October 28: Excision.

October 29: Results for flat modules.

1993-3

November 8: Discussion on the Karoubi operator κ on $\mathcal{C}A$ arising from the Alexander-Spanier differential.

November 27,28: Coquereaux-Kastler lifting of ΩA into $\overline{A}\tilde{A}$.

December 1: On mixed complexes homotopy equivalent to zero. Comment on Karoubi's κ on $\mathcal{C}A$.

December 2: More on Coquereaux-Kastler results.

December 3: Showing that $[b, h]$ induces $-S$ on M/BM when $[B, h] = 1$.

December 15: Note about motivation for B .

December 23,24: Analysis of standard bimodules resolutions of A .

December 25,26: Construction of a section of the surjection of normalized resolutions $\tilde{A} * \mathbb{C}[\epsilon] \rightarrow \tilde{A} * \mathbb{C}[\epsilon]/([\epsilon, e])$.

December 28: Canonical bimodule liftings for $A * \mathbb{C}[h] \rightarrow A * \mathbb{C}[\epsilon]$.

December 27: Key diagram for A -bimodule resolutions.

December 30: More on B -acyclic mixed complexes.

1993-4

Rough notes starting before 7 July on Cuntz's version of Nistor's bivariant Chern character.

1993-5

Rough notes written 21 July - 1 August.

1993-6

Rough notes written 7 August-21 August.

1993-7

Rough notes written 21 August - 25 August.

1993-8

Rough notes written 25 August - 31 August.

1993-9

Rough notes written in September.

Contents 1994

1994-1

- January 1: Calculations to determine maps $f : M \rightarrow \Lambda \tilde{\otimes} \mathcal{B}(\mathbb{C})[1]$ of mixed complexes.
- January 3: Ideas from a paper of Karoubi: A definition of cohomology with arbitrary coefficients in terms of non-commutative differential forms.
- January 5: On the exact sequence $0 \rightarrow \mathbb{C} \rightarrow \Omega G \rightarrow \bar{\Omega} A \rightarrow 0$ of mixed complexes.
- January 6: On the left and right action of $A * \mathbb{C}[\epsilon]$ on $A * \mathbb{C}[h]$.
- January 7: Constructing homotopy inverses to the maps $P\bar{\Omega}\tilde{A} \rightarrow P\bar{\Omega}A$ and $C^\lambda(A) \rightarrow \bar{C}^\lambda(A)$.
- January 13: Applications of Lie and BRS cohomology to cyclic homology.
- January 16: Reduced analogue of the bar construction.
- January 20,22: The Vaughan Jones construction.
- January 24: Notes on retractions, the tensor algebra $T(\overline{A * \mathbb{C}[\epsilon]})$, and a non-commutative version of the derivation ∇ .
- January 25: Brief notes on the derivation ∇ on a graded algebra, and on a unital algebra with a linear retraction.
- January 27,28: Continuing the discussion on a unital algebra with a linear retraction.
- January 29: Review of results for the DG algebra $\tilde{A} * \mathbb{C}\epsilon$ with a homomorphism $T(A^*) \rightarrow T((\tilde{A} \oplus \mathbb{C}\epsilon)^*)$.
- January 30: Homotopy formulas for DG algebras. Summary of formulas for the Vaughan Jones construction.

1994-2

- February 1: Results for $B \subset A \xrightarrow{\rho} B$ where ρ is a B -bimodule map.
- February 3: Review of program.
- February 4,5: Formulas for a connection and a principal bundle.
- February 7-18: Constructing an explicit S operator on $\bar{C}^\lambda(A)$ starting from a retraction $\rho : A \rightarrow \mathbb{C}$.
- February 19: More on the S operator.
- February 20: Can we view HTP as a kind of Laplacian method?
- February 21: Return to the construction of a lifting $\bar{C}_\lambda(A) \rightarrow C_\lambda(A)/C_\lambda(\mathbb{C})$.
- February 22,24: Calculations involving derivations on $A * \mathbb{C}[h]$.
- February 25,26: On using contractions.
- February 27: Two left and right $A * \mathbb{C}[\epsilon]$ module structures on $A * \mathbb{C}[h]$.
- March 2,4: On the standard bimodule resolution $A * \mathbb{C}[h]$ and the tensor algebra $T(A)$.
- March 5,8: Review and summary of recent calculations. List of ideas.
- March 17: Organizing matters arising from a construction of Brodzki.
- March 18: Constructing a vector field on the space of pairs of unitary involutions on a hermitian vector space.
- March 20: Return to considering $\rho : RA \rightarrow R_S A$ with new ideas incorporated.

1994-3

March 23: List of Vaughan Jones formulas.

March 28-April 2: Ideas from Kadeson's and Pimsner's papers related to the Vaughan Jones construction.

April 4: Davydov's result and implications.

April 5: Multipliers and non-unital algebras.

April 8: Perfect algebras and central extensions. Multipliers. Morita equivalences for non-unital rings.

April 9: Good A -modules and good $A \otimes_A A$ -modules.

April 9: Discussion on the standard normalizing resolution for \tilde{A} where A is non-unital.

April 11: Results for a non-unital algebra A , idempotents $e \in A$ and $A' = Ae \otimes_B A$ where $B = eAe$.

1994-4

April 12: Flat A -modules.

April 14,15: On algebras of the form $Ae \otimes_B eA$ where $B = eAe$.

April 17,18: On $V \otimes_B W$. When is an algebra Morita equivalent to a unital algebra? Summary of ideas.

April 21: Reworking Wodzicki's result where $A \subset R$ is a left ideal such that $A = A^2$

April 22: Limits in $\text{Mod}_g(A)$ (good A -modules).

April 23: Existence of sufficient good flat modules. Left derived functors $L_n F$ where $F : \text{Mod}_g(A) \hookrightarrow \text{Mod}(A)$.

April 24: On the book "Torsion Theories" by Jonathan Golan.

April 25,27,28: Results on good A -modules.

April 30: Morita equivalence of rings.

May 1: Generators in abelian categories and Morita equivalences,

May 3: A result of Wodzicki on flat A -modules.

1994-5

May 6: Return to Pimsner's construction.

May 9: Excision.

May 12,25: If a ring A is such that $(1 + A)^\times$ is a group then any simple A -modules is null. Consequences of this result.

May 27: Good A -modules when $A = q\mathbb{Z}$, $q > 1$.

May 28: Let A be an ideal in a non-unital ring R then the restriction of scalars functor $R - g_A \text{mod} \rightarrow A - g \text{mod}$ is an equivalence of categories.

May 29: Good I -modules when $R = k[x_0, x_1]$, $I = (x_0, x_1)$.

June 5: Is $R - g_I \text{mod}$ an abelian category? Examples.

June 9: Good A -modules for a direct sum of rings.

June 12: R -modules as quasicohherent sheaves on $\text{Sp}(R)$. Good' and good modules.

June 13: Studying modules for $R = T(V)$, $I = \bigoplus_{n \geq 1} V^{\otimes n}$, where V is 2-dimensional.

June 14: Results for a finitely generated ideal in a ring: $I = \sum_{i=1}^n Rx_i$.

June 17: Summarising results for $I \subset R$ unital.

1994-6

June 18: Review of previous day's work.

June 19: Additional comments on $D(R/I - \text{mod})$ and $D(R - \text{mod})$, R unital.

June 20: If M is a bounded complex of R -modules then $(R/I) \overset{L}{\otimes}_R M = 0$ iff M is quasi-isomorphic to a complex of good flat modules.

June 21: More on good R -modules.

June 22: Morita equivalences.

June 23: Solid R -modules for $R = T(V)$.

June 25: On the ring $\text{Hom}_{R-\text{mod}/I-\text{nilp}}(R, R)$, $I = I^2$. Examples.

June 26: Relating $\text{Hom}_A(A, A)$ and $\text{Hom}_B(B, B)$ where A is a solid ring, $K \subset A$ is a ring and $B = A/B$.

June 27: Note on a complex M of R -modules such that $M \overset{L}{\otimes}_j R/I = 0$.

June 28: Sheaf situation results.

June 29: Splitting off a contractible complex to obtain a minimal complex for the complex $\dots X^{-1} \rightarrow X^0 \rightarrow X^1 \rightarrow \dots$.

June 30: Proving: if $M/AM = 0$ and $N = 0$ then $\text{Hom}_A(M, N) \xrightarrow{\sim} H_{\mathcal{M}(A)}(M, N)$ and similar results for the "good" tensor product.

July 2: Results for M firm.

July 5,7: On torsion theories. Showing firm modules form an abelian category when R is a noetherian commutative ring.

July 8: About pure exact complexes and pure injective modules.

July 9: Proving $-\otimes_R Q$ is injective in the functor category if Q is pure injective.

July 10: On the flat firm resolution.

July 11: Constructing a flat firm R -modules.

July 12: Pre-additive categories. How to calculate $L_{j!}(j^*M)$.

July 13: Studying the derived category situation: $D_+(\text{mod}(R))$.

July 15: Morita invariance examples.

1994-7

July 16: Let \mathcal{A} be small abelian category then $\text{Lex}(\mathcal{A}, \text{Ab}) \hookrightarrow \text{Add}(\mathcal{A}, \text{Ab})$. Also, localizing subcategories.

July 17: More Morita invariance examples. Review of adjoint functors and results on solid modules.

July 18: Results from Prest's book, particularly on pre-radicals.

July 19: More on the example $R = k[x, y]$, $I = (x, y)$.

July 21: General torsion theory on $\text{mod}(R)$.

July 23: Derived category version of $\mathcal{M}(R, I) = \text{mod}(R)/\text{null}$.

July 24: Direct proof that Morita invariance for the firm derived category.

July 28: Derived category version of Morita equivalence.

July 30,31: How to obtain quasi-coherent sheaves over \mathbb{P}_n from a non-unital ring. Munkholm's talk on the action of the poset \mathbb{N} on a category.

August 1: Puzzle concerning Morita contexts.

August 3: If we don't assume $I = I^2$, then there are equivalences of module categories which are not Morita equivalences?

August 8,9: Derived category picture.

August 10: Checking claims about flat resolutions. Beilinson theory of sheaves on \mathbb{P}^n and improvements.

August 11: Problems on perfect complexes and excision result.

August 12: Beilinson theorem.

August 14: On a Geigle-Lenzing paper on perpendicular categories.

August 25: More category theory results.

August 26: On thick subcategories of a triangulated category. Grothendieck category results. For $\mathcal{A} = \text{mod}(R)$, $\mathcal{I} = \text{tors}(R, I)$, $\mathcal{A}/\mathcal{I} = \mathcal{M}(R, I)$, when is $D^+(R)_{\text{tors}} \xleftarrow{\sim} D^+(\mathcal{I})$?

August 30: Is $M \mapsto 0 \otimes_R M$ fully faithful?

September 1,2: On the derived category Δ , starting with a Grothendieck category \mathcal{A} and localizing

Serre subcategory \mathcal{I} , $\mathcal{I} \xrightleftharpoons[i!]{i_*} \mathcal{A} \xrightleftharpoons[j_*]{j^*} \mathcal{A}/\mathcal{I}$.

September 3: Remarks on a thick subcategory of a triangulated category. The equivalence of triangulated categories $\text{firm} D(R, I) \simeq D^-(\mathcal{M}(R, I))$ when $I = I^2$.

September 5: Candidates for a firm derived category.

September 9,11: On the Morita context $\begin{pmatrix} R & Q \\ P & S \end{pmatrix}$.

1994-8

September 23,24: Explaining the working principle: results which hold when $I = I^2$ hold if $I^n = I^{n+1} = \dots$ for some n , and are likely to hold if a pro-object I^∞ is introduced.

September 25: Showing $(R/I^\infty) \overset{L}{\otimes}_R I^\infty = 0$ implies $(R/I) \overset{L}{\otimes}_R I^\infty = 0$. Applications to Hochschild homology.

September 29: Quillen's proof that $D(\mathcal{A})/D(\mathcal{A})_{\mathcal{I}} \rightarrow D(\mathcal{A}/\mathcal{I})$ is an equivalence of triangulated categories when \mathcal{A} is an abelian category and \mathcal{I} a Serre category.

September 30: Exact sequence $\bigcup_n \mathcal{M}(I/J^n) \hookrightarrow \mathcal{M}(I) \twoheadrightarrow \mathcal{M}(J)$ when $J \subset I$ are ideals in R . Cuntz's excision proof for bivariant periodic cyclic cohomology.

October 2: Review of pro-objects in an abelian category.

October 4: Why the inverse system $n \mapsto W_n = \bigoplus_{k \geq n} E_k$ is projective as a pro-object.

October 5: Correction to a result on faithful functors.

October 7,8: To show that $D^b(\text{mod}(R/I^\infty)) \rightarrow D^b(\text{mod}(R)/(\text{mod}(R/I^\infty)))$ is an equivalence of triangulated categories when I is approximately h-unital.

October 9: Truncations of a complex.

October 10: When is an inverse system in a category essentially constant?

October 13: Summary of pro-derived category results.

October 20: Review $D(\mathcal{A})_{\mathcal{S}} \hookrightarrow D(\mathcal{A} \twoheadrightarrow G(\mathcal{A}/\mathcal{S}))$ where \mathcal{A} is an abelian category and \mathcal{S} a Serre subcategory.

October 21: If A is a C^* -algebra, can we identify $\mathcal{M}(A)$ with something simpler?

October 23,26: Return to an abelian category \mathcal{A} with Serre subcategory \mathcal{S} to prove $K(\mathcal{A}/\mathcal{S})$ is the quotient of $K(\mathcal{A})$ by the thick subcategory $\text{Ker}(K(\mathcal{A}) \rightarrow K(\mathcal{A}/\mathcal{S}))$ consisting of complexes which become contractible in \mathcal{A}/\mathcal{S} . Another proof of $D(\mathcal{A})/D(\mathcal{A})_{\mathcal{S}} \simeq D(\mathcal{A}/\mathcal{S})$.

October 27: Proof that $i_* : D^b(\mathcal{S}) \rightarrow D^b(R)_{\mathcal{S}}$ is fully faithful in the h-unital situation.

October 29: Transitivity result.

1994-9

October 30: If M is a finitely presented R -module then the right adjoint $j_* : \mathcal{M}(R, I) \rightarrow \text{mod}(R)$ (of the quotient function j^*) is given by $(j_*(j^*N) = \varinjlim_n \text{Hom}(I^{(n)}, N)$.

October 31: Calculations for $S = \bigoplus_{n \geq 0} S_n$, an \mathbb{N} -graded ring, and $M = \bigoplus_{n \in \mathbb{Z}} M_n$, a \mathbb{Z} -graded S -module.

November 1: Remarks about $D^+(\text{tors}(R, I)) \rightarrow D^+(R)$ being fully faithful.

November 3: Remarks on modules over a h -unital ring.

November 6: Old problem: to construct an infinite general linear group out of $\text{Aut}(P)$ for $P \in \mathcal{P}(A)$, the category of finitely generated projective A -modules.

November 9: Lundell review.

November 10: Discussion of the localization of M for the torsion theory $\text{tors}(R, I)$.

November 17: Correction to the proof of a result proved in October 31 entry.

November 18: On $\mathcal{Z}(M, N) = \{Z \subset M \oplus N \mid \rho_1 : Z \rightarrow M \text{ is an } \mathcal{S} \text{ isomorphism}\}$.

November 22: TTF theories.

November 24: Suslin's pseudo-free resolutions.

November 28: Results following $\mathcal{S} \xrightarrow{i_*} \mathcal{A} \xrightarrow{j^*} \mathcal{A}/\mathcal{S}$ and the assumption that i_* is fully faithful.

November 30, December 1: Cyclic homology theory and K -theory associated to a ring where $A = A^2$.

December 2: On the functor $\mathcal{A} \rightarrow \text{mod}(R^{op}) : M \mapsto \text{Hom}_{\mathcal{A}}(U, M)$, where \mathcal{A} is an abelian category and U an object of \mathcal{A} .

December 3,4: Gabriel-Popescu theorem.

December 6: On Roo's theorem characterizing abelian categories of the form $\mathcal{M}(A)$ where $A = A^2$. Extending the construction of assigning Hochschild homology groups to a Roos-type abelian category to cyclic homology and K -theory.

December 8: Review of relations among $K_1(R/I) \xrightarrow{\partial} K_0(I)$, Atiyah-Singer map for invertibles in the Calkin algebra to the restricted Grassmannian, and the Cayley transfer picture of the Grassmannian.

December 11: Morita equivalence for d -firm perfect complexes.

December 14,15: Constructing a free resolution of a right-bounded complex of R -modules.

December 16,19: On complexes of R -modules such that $\mu : I \otimes_R U \rightarrow U$ is a homotopy equivalence. More on Morita equivalences.

December 20,21: On projective D -firm complexes.

December 28: Constructing an idempotent ring A such that $A \otimes_A \otimes_A \rightarrow A \otimes_A$ is not an idomorphism.

December 28: More results using Morita contexts $\begin{pmatrix} R & Q \\ P & S \end{pmatrix}$, ideals $I \subset R$, $J \subset S$ such that $PIQ \supset J^s$, $QJP \supset I^t$, some $s, t \in \mathbb{N}$. These results include $I \otimes_r I \otimes_R = J \otimes_S J \otimes_S$ and $I^\infty \otimes_R / I^\infty \otimes_R \xrightarrow{\sim} J^\infty \otimes_S J^\infty \otimes_S$.

Contents 1995

1995-1

- January 10: To define K_1 for non-unital idempotent rings.
- January 11: Factoring a Morita equivalence $\mathcal{M}(A) \xrightarrow{\sim} \mathcal{M}(B)$ into simpler steps, when A and B are firm.
- January 13: Calculating $[M_n(A), M_n(A)]$, $n \geq 2$. A mechanism for Morita invariance of K_1 . The canonical map $P \overset{L}{\otimes}_A A \overset{L}{\otimes}_A B \rightarrow B$ is a quasi-isomorphism $\Leftrightarrow B = P \otimes_A Q$ is h-unital.
- January 14-February 6: On the Morita invariance of $K_1(A)$.
- February 16: From Kucerovsky's thesis (Cohen's theorem for C^* -algebras and Hilbert modules).
- February 18,19: Understanding Kucerovsky's functional calculus for unbounded regular normal operators.
- March 8: Notes on the book of Jensen and Thomsen. Ideas. Notes on KK -theory.
- March 10: Summary of facts for polar decomposition.
- March 16: Notes on C^* algebras from Pedersen's book.
- March 25: More on Morita contexts. Problems to be solved, including Morita invariance for higher K groups.
- March 27,30: Notes on complexes of finitely generated projective R -modules and K -theory.
- April 2: A different proof of Milnor's result: $\mathcal{P}(\tilde{A}) \rightarrow \mathcal{P}(R) \times_{\mathcal{P}(R/A)} \mathcal{P}(\mathbb{Z})$ is an equivalence ($\mathcal{P}(R)$ denotes finitely generated projective R -modules).
- April 5-18: Results and calculations for a complex U where the identity operator has a deformation $1 - [d, h]$ which is nuclear. Analysis of homotopy idempotents.

1995-2

- April 20-25: Continuing the study of a complex where the identity has a nuclear deformation $1 - [d, h]$. Homotopy equivalences.
- April 29-May 2: Comments on previous work and further related results.
- May 4,5: An inductive construction which refines a homotopy idempotent to an A^∞ idempotent.
- May 6: Calculation with a Morita context.
- May 11: Length 1 complexes.
- May 16: Results for the category of strictly perfect homotopy form complexes.
- July 1: Comments on obtaining the higher K -theory groups from the category of perfect firm complexes.
- July 18,19: Statement of program for obtaining relations between Pedersen-Weibel delooping, cone and desuspension of a ring and John Roe's finite propagation C^* algebras.
- July 28, August 7: On the Grothendieck groups $L_n(R, A)$ generated by finitely generated projective R -module complexes supported on degrees $0 \leq k \leq n$ which become contractible modulo A .
- August 26,28: On establishing Morita invariants for Hochschild homology of h -unital rings.

1995-3

August 29: Copy of Quillen's paper: K_0 for non-unital rings and Morita invariance. This paper is published as *J. Reine Angew. Math.*, **472**, 1996, 197-217, doi:10.1515/crll.1996.472.197, and is reproduced here with the kind permission of the Editor.

1995-4

September 5,6: Reducing Morita invariance to simplest steps.

September 8: Inductive limits in a Roos category $\mathcal{M}(A)$, A firm.

September 14: Proving that if A is idempotent, $K \subseteq A$ is an ideal such that $KA = 0$ and $B = A/K$, then $AK = A \Leftrightarrow K_1(A) \xrightarrow{\sim} K_1(B)$.

September 15: Results for a ring B , a B -module surjection $A \twoheadrightarrow B$ and an ideal $K \subset A$ such that $KA = 0$.

September 16: Example.

September 22: Morita invariance for cyclic homology on h -unital rings.

September 24: Multiplier algebras.

October 1: A difficult exercise with adjoint functors and adjunction maps.

October 7,8: Higgin's thesis on Leibniz algebras. Dialgebras.

October 15: Direct approach to Morita invariance of K_1 for firm rings.

October 19,20: Attempt to define HH and HC intrinsically for a Roos category.

October 21: Proof that for A an ideal in ring B and M a B -module, $A \otimes_B M \xrightarrow{\sim} M \Rightarrow B \otimes_B M \xrightarrow{\sim} M$.

1995-5

October 24: Bimodule approach to HC .

October 5-November 9: Further calculations related to recent work.

November 26: Proof that for $\begin{pmatrix} A & Q \\ P & B \end{pmatrix}$ a completely firm unital context over a ground ring k and A a h -unital and k -fit ring, B is h -unital iff $P \overset{L}{\otimes}_A \overset{L}{\otimes}_A Q \rightarrow B$ is a quasi-isomorphism.

December 1: More on multipliers and Morita contexts.

December 7: Record of observations made while working on ring homomorphisms which induce Morita equivalences.

December 8: New ideas for handling whether a ring homomorphism induces a Morita equivalence.

December 10: More results on Morita contexts.

December 17: Discussion of the problem of defining iterated tensor products of bimodules.

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Contents 1996

1996-1

January 3,4: On equivalence of categories.

January 12: Formulas involved in the equivalences:

$$\begin{array}{ccc}
 \mathrm{mod}(\mathbb{R}) & \begin{array}{c} \xrightarrow{\mathrm{extn}} \\ \xleftarrow{\mathrm{res}} \end{array} & \mathrm{mod}(\mathbb{H}) \\
 \downarrow \sim & & \parallel \\
 \mathrm{mod}(\mathbb{C}[\sigma]_+) & \begin{array}{c} \xrightarrow{H \otimes -} \\ \xleftarrow{H \otimes -} \end{array} & \mathrm{mod}(\mathbb{C}[\sigma]_-)
 \end{array}$$

April 4: A six term sequence of kernels and cokernels.

May 24: Canonical resolutions over P^1 .

August 1-16: Morita invariance of K -theory for h -unital rings.

1996-2

Quillen's paper: Module theory over non-unital rings.

Contents 1997

1997-1

(First 7 pages missing.)

February 2-14: Informal working notes, preparation for lectures and the second chapter of a paper on Morita invariance for K_* of rings equivalent to unital rings.

1997-2

February 14-22: Informal working notes on Morita invariance for K_* .

1997-3

February 23-March 4: Informal working notes on Morita invariance for K_* .

1997-4:

March 5-10: More informal working notes for K_* including notes on Lie algebra homology of $\mathfrak{gl}(A)$, Dennis trace and cyclic homology.

1997-5:

March 23-April 27: Miscellaneous notes including some on stability for $GL_n(F)$, F a field.

1997-6

Probably written in July: Working notes in preparation for Chapter 2 of Morita invariance of K_* .

1997-7

Undated continuation of notes written in previous file.

1997-8

Continuation of notes in previous two files.

1997-9

Undated: Review of Geigler-Linzing existence of adjoints. Gelfand's results and quivers.

1997-10

Written in September: Continuation of work in previous file, includes some on applications of quivers and an introduction to the Toeplitz algebra.

1997-11

Undated. Contains material on Cuntz's talk on group algebras and C^* algebras.

1997-12

November 13: Proof that $K_0(A) \xrightarrow{\sim} K_1(C)$ where $C = \begin{pmatrix} A & Q \\ P & B \end{pmatrix}$ is a Morita context such that $PQ = B$.

November 24: Notes on ideas inspired by Pimner's treatment of the Toeplitz algebra.

November 30: Notes on an algebraic treatment of Kasparov theory.

December 3: If $U : U_1 \xrightarrow{d} U_0$ is a complex of R -modules where the identity map can be deformed to a nuclear map then U is homotopy equivalent to a length 1 complex of finite projective modules.

December 7: Two proofs that given $\begin{pmatrix} A & Q \\ P & B \end{pmatrix}$ such that $QP=A$ and $PQ=B$ then $K_0(A) \simeq K_0(B)$.

December 14: Cosheaves over a tree.

December 22: Carey-Evans result that $U(n, 1)$ acts naturally on the Cuntz algebra \mathcal{O}_n .

1997-13

October 26-November 22: Stability results for $GL(F)$ where F is a field. More on Morita invariance of K_0 . Suslin's excision result. Cyclic homology $C_*(\mathfrak{gl}(A))$.

1997-14

November 25-December 3: Informal working notes including some on the Toeplitz algebra, and on understanding $K_0(A[T])$.

1997-15

December 14- : Notes on many topics including the Atiyah-Bott proof of the periodicity theorem, fibering over a circle and Morita invariance.

1997-16

Undated mixed notes on Morita invariance of K_* .

1997-17

December 14-22: Notes on many ideas including an observation that plumbing with coaxial cable might be like surface theory.

1997-18

December 26,27: LC network circuit theory. Idea: Analogy between circuits and surfaces, n-port corresponds to a surface with n-boundary components. Program is to construct interesting self-adjoint operators by gluing connecting ports together might have a surface analogue. Should enter by Konsevich link-Euler characteristics of surface graphs given by ζ values.

December 28: Review relevant Hodge theory and circuit theory. Polarized Hilbert spaces.

December 29: LC circuits and $V \oplus V'$, potentials and currents.

December 30: Review of previous few days work.

December 31: Developing the port picture.

Contents 1998

1998-1

January 1: LC circuits. Polarized real symplectic spaces. Complex structures and symplectic structures. Forced harmonic oscillators.

January 2,3: Calculations with symplectic and complex structures. Hamiltonians and polarizations. Compressed harmonic operator. Forced harmonic operator.

January 4,5: Response of forced harmonic operator.

January 6: Coupling to a massive harmonic operator. Coupled harmonic operators.

1998-2

January 7: Coupling of two oscillators.

January 8: LC circuits review. Coupling. Partially defined maps and correspondances. Electrical networks.

January 10: Partial isometries. Decomposition of K -modules.

January 11,12: Partial isometries and response functions. Graph analysis and L^2 1-chains. K -modules. Scattering. Infinite determinants.

January 14: Coupling matrix. Response function for a partial isometry of codim 1. Scattering.

1998-3

January 15: Linking scattering theory and partial isometries. Examples of scattering. Discrete wave equation. Review.

January 17: Lagrangian type harmonic oscillator. LC circuits and partial isometries, partial unitaries and scattering.

January 18: Scattering operator for a partial isometry. LC circuit with capacitances at vertices and inductances for edges on the $SL_2(\mathbb{Z})$ tree.

January 19: Lecture preparation.

January 20: Partial notes for lecture.

1998-4

January 20: Stable isomorphisms on finite dimensional Hilbert spaces.

January 21: Spectrum of circuit over the $SL_2(\mathbb{Z})$ tree. Notes on a paper on Novikov-Morse theory. Return to tree \mathbb{R} acted on by \mathbb{Z} with 2-ports as edges and coupling at vertices.

January 22: Complex version of the tree.

January 23: Analysis of Euclidean space V with a positive definite quadratic form (v, Av) and subspace W . Polarizations. Working with the GNS philosophy. Connecting LC response to K -modules. Finding a Hilbert space interpretation of an n -port which leads to a partial isometry of codimension r and hence to a r -dimensional vector bundle over S^2 .

January 25: Review of problem.

1998-5

January 25: Pencils of hyperplane sections in Lefschetz theory relating to 2-ports.

January 26: Review of puzzle. Considering a reflection of a 1-port coupled to a transmission line.

January 27: Scattering related to n -ports and transmission lines.

January 28: Restating problem: given an LC 1-port, find an associated partial isometry - alternative approaches.

January 29: Statement of previous day's progress - obtaining a vector bundle over \mathbb{P}^1 from a response function. Moment problem. Restatement of problem.

January 30: Self-coupling. Symplectic quotient with a Lagrangian subspace. Metaplectic group.

1998-6

February 1,2: Constructing a resolution for bundles $F \subseteq \mathcal{O} \otimes (V \otimes V^*)$ over \mathbb{P}^1 where each $F_s = \begin{pmatrix} Z_s \\ 1 \end{pmatrix} V^* \subseteq V \oplus V^*$ is a Lagrangian subspace and $V = U/W$ is a subquotient of $H = H^+ \oplus H^-$. Symplectic quotients.

February 3: Constructing a Hilbert space from data coming from a response function Z .

February 4: Review Jacobi matrix theory and associated de Branges work.

February 5: Relationship between continued functions and moment problem.

February 6: Review ideas on continued fractions.

February 8: Return to symplectic picture with harmonic oscillator and symplectic coupling. Understanding successive coupling arising from continued fractions. Continuous limit of ladder network.

February 10: An n -fold transmission line. Scattering. Connecting an r -port to an r -fold transmission line transforms the impedance Z to its Cayley transform $S = \frac{Z-1}{Z+1}$. Analysis of S .

February 11: Review of recent work.

1998-7

February 12: Preparation for lecture on the K -module associated to a scattering operator $S(s)$, analytic on a neighbourhood of the closed disc, unitary on the boundary.

February 13: More on scattering operators and associated vector bundles. Factorization of S .

February 14: Eigenvalue picture and associated calculations.

February 16: Review partial isometry $X^d \xrightarrow[a]{b} Y^{r+d}$, $\|ax\| = \|x\| = \|bx\|$, $a^*a = 1$, $b^*b = 1$. Eigenvalue equation. Resolvent calculations.

February 19: Brief note on scattering.

February 20: Space X together with contracting operator u determines non-trivial scattering.

1998-8

February 21: Link between $U(p+q)$ and $U(p,q)$. Simple 2-ports. Transfer matrix.

February 22: Coupling transmission line segments in a translation invariant way. Coupling invariantly over the $PSL_2(\mathbb{Z})$ tree. Examples.

February 26: 3-ports and 2-ports. Examples.

February 27: More on coupling. 2-ports with $\mathbb{Z}/2$ symmetry.

1998-9

February 28-March 3: Review and continuation of recent calculations.

March 6: Constructing eigenvector as series.

March 10: On $\begin{pmatrix} \phi_0^+ \\ \phi_0^- \end{pmatrix} = \begin{pmatrix} \frac{1}{\lambda a} & \frac{b}{a} \\ \frac{b}{a} & \frac{\lambda}{a} \end{pmatrix} \begin{pmatrix} \phi_0^+ \\ \phi_0^- \end{pmatrix}$, $b \in i\mathbb{R}$, $0 < \frac{b}{i} < 1$, $a = \sqrt{1 + |b|^2}$. The Γ -graph $S = \left(\frac{\lambda^{-1}S^2 + b}{bS^2 + \lambda} \right)$. Analysis of $w \mapsto z = \frac{w-b}{w^2(1-bw)}$.

March 11,12: Continue study of $z = \frac{1}{w^2} \frac{w-b}{1-bw}$ and related partial isometry.

1998-10

March 13: Continuing the analysis with $|S(\lambda)| \leq 1 - \epsilon$ for $|\lambda| < 1$. Studying the linear graph of a 2-port.

March 14: On $\ell^2(\Gamma)$, $\Gamma = PSL_2(\mathbb{Z})$.

March 15: Spectral theory and Green's function $(\lambda - u)^{-1}$.

March 16: Solving $\frac{\lambda + \lambda^{-1}}{a} = z + z^{-1}$. More on periodically coupled 2-ports.

March 19: Hilbert space picture corresponding to the partial fractions expansion $S(z) = \begin{pmatrix} 1 & h_0 \\ h_0 & 1 \end{pmatrix} \begin{pmatrix} Z & 0 \\ 0 & 1 \end{pmatrix} \dots$ of analytic $S(z)$ in $|z| < 1$.

1998-11

March 21-27: Review putting together work on partial isometry $X \xrightarrow[b]{a} Y$, contraction operator $\gamma : X \rightarrow X$ and operator $S(\lambda) : V^- \rightarrow V^+$.

March 28,29: Orthogonal polynomials in $L^2(S^1, d\mu)$.

1998-12

March 30,31: More on orthogonal polynomials in $L^2(S^1, d\mu)$.

April 2-9: A contraction $\gamma : X \rightarrow X$ gives rise to a partial isometry $X \xrightarrow[b]{a} Y$ such that $b^*a = \gamma$, $\overline{aX + bX} = Y$.

1998-13

Mainly undated. Numbered 1-58 with page 32 dated October 2, 1998, page 50 dated October 5, 1998.

Continues calculations related to a partial isometry $X \xrightarrow[b]{a} Y$, $aX \oplus V^+ = V^- \oplus bX = Y$ of type $\mathcal{O}(n)$.

1998-14

Undated, numbered 5-120. Continues calculations. Mentions de Branges formulas, reflection positivity, invariant approach, LC network, configuration space and hermitian versions.

1998-15

Undated, numbered 121-201. Continues calculations. Mentions quaternionic version of orthogonal polynomials, understanding how de Branges goes from a scalar product on a space of polynomials of degree $< n$ to an isometric embedding in a Hardy space. Atiyah's divisors. Bargmann kernel, nested circle argument.

1998-16

Undated, numbered 202-249. Continues calculations. Mentions zeroes of a scattering function.

1998-17 Undated, numbered 250-349. Continues calculations. Mentions working with orthogonal polynomials, Hardy spaces, reproducing kernels.

Contents 1999

1999-1

January 18,19: Notes of lectures on operators and related analytic functions.

January 25: Riemann sphere. Fractional linear transformations. Hyperbolic plane. Schur expansion for $f \in H^\infty$, $\|f\|_\infty \leq 1$.

1999-2

May 14: Models for emission and absorption of radiation given by simple aharmonic oscillator coupled to a photon field: string model, transmission model.

May 16: Examples.

May 17: Simple harmonic oscillator review.

May 21: Review QFT's with one dimensional space.

May 25: Quantizer examples of wave equations.

May 27: Harmonic oscillator algebra.

May 31: Ideas from scratch pad.

June 4: Wave equation for a light string attached to a simple harmonic oscillator.

June 6: Green's function for a second order differential equation. Sturm-Liouville equation.

June 9: Philosophy of harmonic oscillators in infinite dimensions.

June 10: Recalling classical and quantum treatment of the harmonic oscillator (finite dimensional).

June 11: Ideas from scratch book.

June 14: Fermion quantization.

June 21: Partition function which gives the triple Jacobi product identity.

June 28: Infinite chain of coupled pendulums. $\ddot{u} = u_{n+1} - 2u_n + u_{n-1}$.

July 18: On inverse scattering.

July 23: Examples of orthogonal polynomials on S^1 .

July 26: List of points from scratch work.

August 28: On the discrete one-dimensional Dirac equation $\begin{pmatrix} p_n \\ q_n \end{pmatrix} = \frac{1}{k_n} \begin{pmatrix} 1 & h_n \\ \hbar_n & 1 \end{pmatrix} \begin{pmatrix} zp_{n-1} \\ q_{n-1} \end{pmatrix}$,
 $n \in \mathbb{Z}$.

August 29: Two-sided scattering where (h_n) has finite support.

September 6-11: Scattering for a disc.

September 15,16: Formulas relating 2×2 transfer and scattering matrices.

1999-3

Informal undated notes numbered 1-65 by Quillen on material related to the harmonic oscillator and continuous grid situation.

1999-4

Informal undated notes numbered 66-129 on grid space.

1999-5

Informal undated notes numbered 130-174. On page 162 Quillen discusses Birkoff factorization of the S matrix.

1999-6

Notes numbered 175-247.

1999-7 Notes numbered 248-290, includes notes for a lecture.

1999-8

Notes numbered 291-371.

1999-9

Notes numbered 371-541.

1999-10

Notes numbered 542-619

1999-11

Notes numbered 620-695. Continuing with grid analysis.

1999-12

Notes numbered 696-790. Described grid space as a space of meromorphic functions inside $L^2(\mathbb{R}, \frac{d\xi}{2\pi})$ Mentions Irving Allen: Phylotaxis, $PSL(2\mathbb{Z})$ tree, acyclic coefficient systems over trees.

1999-13

Numbered 806-910 (pages 791-805 missing).

1999-14

Numbered 911-1000.

1999-15

Undated, numbered 390-447 (pages 1-389 missing). Mentions Pick function, positive harmonic functions, measures, scattering function $S(z)$.

1999-16

Numbered 448-500.

1999-17

Numbered 501-570.

1999-18

Numbered 575-633.

1999-18

Numbered 571-633. Lecture 5.

1999-19

Numbered 634-675. Mentions mixed Hodge structure for surfaces, curves over a finite field, algebra of correspondances over a curven intersection pairing, Hardy spaces, positive energy representations of the loop group.

Contents 2000

2000-1

Pages 1-71 missing.

January 29,30: Definition of a grid space. Infinite constant coefficient grid spaces.

January 31: Continuous limits of the constant h grid.

February 1: More on grid space.

February 27: Inverse scattering in a Hilbert space setting.

March 16: On the Dirac equation $\partial_t \phi = \begin{pmatrix} \partial_x & i\hbar \\ i\hbar & -\partial_x \end{pmatrix} \phi$, $\phi = \begin{pmatrix} \phi^1(x, t) \\ \phi^2(x, t) \end{pmatrix}$. Cauchy problem.

March 17: Linear algebra related to Wronskians and quantization.

March 24,25: Harmonic oscillator algebra.

May 13,14: Bessel functions.

May 16-21: Continuous analogue of discrete grid space.

May 23: A derivation for Szegő relations for orthogonal polynomials.

June 18: Wave equation.

June 20: Green's functions for the discrete case.

2000-2

Pages 1-61. Undated. Return to scattering theory and analysis in $L^2(S^1, d\mu)$.

2000-3

Pages 62-138. Undated. More scattering theory calculations.

2000-4

Pages 139-225. Scattering theory calculations. Birkhoff factorization as an integral equation. Green's function.

2000-5

Pages 226-299. Undated. Green's function $G_\lambda(x, y)$ satisfying $(\partial_x - V_\lambda(x))G_\lambda(x, y) = \delta(x - y)$. Dirac equation. Transfer matrix.

2000-6

Pages 300-377. Undated. More on Birkhoff factorization. $\mathcal{O}(-1)$ calculations. Wave equation calculations. Review (page 345).

2000-7

Pages 378-446. Undated. Green's function. Philosophy (page 383).

2000-8

Pages 447-505d. Undated. Descent in grid space. Eigenfunctions. Grid space as a module over $\mathbb{C}[u, u^{-1}]$. Green's function calculations. Abstract framework.

2000-9

Pages 506-555. Undated. On $K_*(\mathbb{C}[\Gamma])$. Review of Cuntz construction.

2000-10

Pages 556-611a. Undated. Cross products $\mathcal{E}_{\Sigma_F}^{ab} \rtimes \Gamma$ and $C(Y) \rtimes \Gamma$.

2000-11

Pages 607b-639. First page dated September 1. Cuntz construction and related material.

2000-12

Pages 640-678. Undated. Matter related to Cuntz construction. Morita equivalence and groupoids.

2000-13

Pages 679-719. Undated. More on the assembly map for \mathbb{Z} and the general $K_*(\mathbb{C}[\Gamma])$. Conjecture for $\Gamma = \mathbb{Z}$.

2000-14

Pages 720-793. Undated. Baum-Connes conjecture. $KK_*(C(B\Gamma), \mathbb{C}) \dashrightarrow KK_*(\mathbb{C}, C_1^*(\Gamma))$?

2000-15

Pages 749-829. Undated. More on $K_*(\mathbb{C}[\Gamma])$.

2000-16

Pages 830-890. Undated. More on $K(\mathbb{C}[\Gamma])$.

2000-17

Pages 891-948. Undated. Cuntz's \mathcal{E}_{Σ_F} Morita equivalence of $C_c(Y) \rtimes \Gamma$ and $C(X)$ where $X = Y/\Gamma$.

2000-18

Pages 949-1000. Undated. Continuing with analysis associated to a Γ action and a partition of unity indexed by Γ .

Contents 2001

2001-1

Brief notes of delivered at a conference in Bonn including notes of Quillen's lecture titled Morita Invariance in Higher K-theory of h -algebras. De Branges functions with infinitely many zeroes. Cuntz's C^* Morita equivalence.

2001-2

May 24: Γ -graded vector spaces and algebras.

May 28: Equivalence of categories of pointed sets and set-like coalgebras.

June 10: Multiplier algebra of an algebra.

June 18,20: Property $e^2 = e$ in ring B gives Morita context $\begin{pmatrix} eBe & eB \\ Be & B \end{pmatrix} \subset M_2(B)$.

July 18: Cuntz's Morita equivalence for groupoids.

2001-3

Quillen's index for 2001.

Pages numbered 1-37, mostly undated.

January 16: Review of problem concerning Morita equivalence of $C_c(\mathbb{R})$ and $C_c(\mathbb{R}) \rtimes \mathbb{Z}$. Linear equation criterion for flatness. Local left units.

2001-4

Pages numbered 38-92. On $\mathbb{C}[\Gamma] \otimes B$ and $B \otimes \mathbb{C}[\Gamma]$ where B is a graded algebra. Note on how a tensor product of Γ -graded algebras is undefined. On a ring with generators h and k such that $kh = h$. On GNS. On positive functionals on a non-central C^* algebra. On Γ -graded algebras.

2001-5

Pages numbered 93a-148. More on Morita equivalence. On a ring with generators e and h such that $e^2 = e$ and $eh = h$.

2001-6

Pages numbered 149-188. Examples of two Morita contexts.

2001-7

Pages numbered 189-242. Further calculations with Morita contexts. Universal rings.

2001-8

Pages numbered 243-299. More on Morita equivalence. Comments on Pedersen-Weibel paper and lower- K -theory.

2001-9

Pages numbered 300-347. More on a Morita context. Progress extending Γ -graded algebras to Morita contexts.

2001-10

Pages numbered 374a-442. More on Morita contexts.

2001-11

Pages numbered 443-486. Morita contexts and Morita equivalence.

2001-12

Pages numbered 487-539. Classifying spaces and assembly maps for a groupoid.

2001-13

Pages numbered 540-604. Categories and groupoids. Cuntz's groupoid

2001-14

Pages numbered 605-689. More on an assembly map for a groupoid, particularly M_2 .

2001-15

Pages numbered 690-715. Calculations associated to the groupoid M_2 . Tensor category for bialgebras.

2001-16

Pages numbered 745a-834. Contains pages dated December 2001. Finite duration impulse response. Interpolation formula. Poisson summation formula. Review of grid spaces.

Contents 2002

2002-1

Pages numbered 835-860.

List of ideas and ways to proceed. Action of $SL(2, \mathbb{Z})$ on $\mathbb{R}^2/\mathbb{Z}^2$ lifted to a line bundle of degree 1.

2002-2

Pages numbered 861-931.

Proof of Serre's theorem on the equivalence of finitely generated projective modules over $C(X)$ and vector bundles over X . Constructing a complex line bundle of degree 1 over \mathbb{T}^2 as a homogeneous space of the Heisenberg group K by \mathbb{Z}^3 .

2002-3

Pages numbered 941-1000.

January 27-February 17: On the trivial line bundle on \mathbb{R}^2 with connection $d + 2\pi i dy$ and an $SL(2, \mathbb{R}) \rtimes \mathbb{R}^2$ action

2001-4

Pages numbered 23-56.

January 16: Notes on Serre's theorem about the equivalence of categories of finitely generated projective modules over $C(X)$ and complex vector bundles over X . Heisenberg group and Lie algebra.

January 22: Connections on a vector bundle over \mathbb{T}^2 .

January 24, February 7: On $\mathcal{L} = \{\phi(x, y) \in C^\infty(\mathbb{R}^2) : \phi(x, y) \text{ has period 1 in } y, e^{2\pi i xy} \psi(x, y) \text{ has period 1 in } x\}$.

February 23, 24: Central extensions of an abelian group.

February 25: Discussion of central extensions specialising to elementary abelian 2-groups.

February 28: Smooth functions on a \mathbb{T}^1 -principal bundle and Toeplitz algebra. Two categories of Γ -modules \mathcal{W} and \mathcal{V} associated to a finite subset of Γ .

March 19, 25: Calculations with $\Gamma = \{1, \epsilon\}$, $\epsilon^2 = 1$, and the Cayley transform.

March 29: Two proofs that if $u : R \rightarrow S$ is a map of unital rings such that the restriction of scalars functor $u^* : \text{Mod}(S) \rightarrow \text{Mod}(R)$ is an equivalence of categories, then u is an isomorphism.

2002-5

Pages numbered 1-43.

February 17-27: More on a line bundle over \mathbb{T}^2 with curvature $2\pi i dx dy$ and the action of the Heisenberg group.

2002-6

Pages numbered 44-113.

March 2-5: Review Heisenberg group.

March 5, 6: Cuntz's M_2 theorem.

March 7: Monoid Γ and reduced monoidal algebra $\mathbb{C}\Gamma = \mathbb{C}\Gamma_+/\mathbb{C}(0)$.

March 8: Calculations with $\Gamma = M_n$.

March 9-11: Cuntz's R construction. More on the principal \mathbb{T}^2 bundle over \mathbb{R}^2 with curvature $2\pi idxdy$. Adjoining an idempotent to the Cuntz algebra $\Lambda * \Lambda$ with Fedosov product. Retracts and Cuntz's RA construction.

March 12: Calculations in $\mathbb{C}[\epsilon] * \mathbb{C}[F] = \mathbb{C}[F\epsilon] \rtimes F$.

March 13: More on a Morita context, an M_2 -graded algebra.

March 14: On a Morita context, an $M_2 \times \Gamma$ -graded algebra.

March 16-19: Representations of the dihedral group, $\mathbb{Z}/2 \rtimes \mathbb{Z}$.

2002-7

March 20-28: On two categories \mathscr{W} with objects $W = \begin{pmatrix} W_+ \\ W_- \end{pmatrix}$, a $\mathbb{Z}/2$ graded vector space equipped with an odd operator X such that $\epsilon X \epsilon = -X$, and \mathscr{V} of vector spaces with two projections p_{\pm} .

March 27-30: Morita context for a group Γ .

March 30: Review Morita equivalence.

April 1: Idea from Serre's theorem. Return to calculations with the line bundle over \mathbb{T}^2 and calculations with $f \in \mathscr{S}(\mathbb{R})$.

2002-8

Further calculations with the principal \mathbb{T} -bundle associated to the Heisenberg group. Understanding "geometric quantization" in d degrees of freedom.

April 20-22 Phase space with 1 degree of freedom. Review of symplectic concepts.

April 23: Category of affine spaces. Reference to Novikov's metal talk ideas. Contact transformations. A solution to the Lagrange equation yielding a family of paths.

April 28: Configuration space equals affine line \mathbb{R} equipped with translational symmetries.

2002-9

June 20-23. LC circuits.

June 25,26: Analysis of $V \rightarrow \begin{matrix} H_+ \\ \oplus \\ H_- \end{matrix} \rightarrow H/V$. Transmission line equation. More on LC circuits.

July 1,3: Philosophy of LC circuits.

July 4,5: Linking partial unitary operators $X \overset{a}{\underset{b}{\rightrightarrows}} Y$, $aa^* = b^*B = 1$ with abstract LC circuits.

July 8: Understanding the representation $SG(T) \rightarrow SC(W^{\perp}/W)$, $L \mapsto ((L + W) \cap W^{\perp})/W$ where $SC(T)$ is the symplectic Grassmannian, T is symplectic, W is isotropic in T , W^{\perp}/W the associated symplectic quotient.

July 10-20: LC circuits and partial isometry operators.

August 9-11: Algebraic picture of GNS; Hilbert space picture of GNS.

August 13,14: Gluing two Hilbert spaces via a contraction. Dilation language.

August 19,25: Example of Thèvenin's theorem. A closed LC circuit.

2002-10

Numbered $a' - c'$, $d - z$.

Symplectic picture for LC circuits.

2002-11

Numbered $a' - z'$, $\alpha - \omega$, $\alpha' - \omega'$.

How to deal with partial dynamics. Review of LC circuits.

2002-12

Numbered $\alpha'' - \omega''$, $\alpha''' - \omega'''$, $a'' - z''$.

More on LC circuits and associated ideas. New quantization for harmonic oscillator.

2002-13

Numbered $a''' - z'''$.

Calculations related to LC networks.

2002-14

Numbered $a - c, c', d - f$, $A - Z$.

On an old question of constructing a Morita context linking \mathscr{W} and \mathscr{V} . Assembly construction. Serre's construction. Example of Wheatstone bridge/ LC networks.

2002-15

Numbered α, β then $\alpha - \omega$.

More on LC circuits.

2002-16

Numbered $\alpha 1 - \omega 1$, $\alpha 2 - \omega 2$.

Thèvenin theory.

2002-17

Numbered $\alpha 3 - \omega 3$.

R networks.

2002-18

Numbered $\alpha 4 - \omega 4$, $\alpha 5 - \omega 5$.

Last page includes material on the Inverse Cayley transform 2003.

2002-19

No numbering. Mixed papers including material on LC circuits and partial unitaries.

2002-20

No numbering. Mixed papers recalling previous ideas and calculations.

2002-21

No numbering. Mixed papers recalling previous ideas and calculations.

2002-22

Numbered 330-389 but starts with a list of references.

Notes on spectral theory of operators, analytic functions on the unit disc, Poisson kernel, positive harmonic functions, Pick functions associated to measures. Review of equivalence of contractions, partial unitaries and scattering operators.

2002-23

Pages numbered 30-37, 45-110, numbers circled.

Calculations associated to Inverse Scattering Theory.

Contents 2003

2003-1

Pages numbered 1-56.

LC networks, Hodge decomposition and K -theory. Discussion of hermitian K -theory. Review of aspects of K -theory.

2003-2

Pages numbered 57-109.

More on hermitian and symplectic K -theory. Review of $Sp(n)$, $SO(2n)$ and symmetric spaces.

2003-3

Pages numbered 110-170.

More on various symmetric spaces.

2003-4

Pages numbered 171-228.

Notes on Cayley transform. LC networks and the Lie algebras $\mathcal{LSP}(2n)$, $\mathcal{LO}(2n)$. Symplectic spaces and polarizations.

2003-5

Pages numbered 229-315.

More on topics in 2003-4, then material on symmetric and skew-symmetric forms on a Hilbert space. Basic representation $H(V) = \begin{bmatrix} V \\ V \end{bmatrix}$ where V is a positive hermitian space with symmetry group $U(n)$ and $H(V)$ has symmetry group $Sp(2n)$.

2003-6

Pages numbered 316-369.

More on $H(V)$ when $V = \mathbb{C}$.

2003-7

Pages numbered 370-450.

More on $Sp(2n)/U(n)$. Spectral flow. Hyperbolic symmetric spaces.